EFP 2.0: A Multi-Agent Epistemic Solver with Multiple e-State Representations: Supplementary Documentation

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The following document provides supplementary information for the paper "EFP 2.0: A Multi-Agent Epistemic Solver with Multiple e-State Representations" submitted to 30^{th} *International Conference on Automated Planning and Scheduling* (ICAPS 2020).

In Section A we will demonstrate the properties of $m\mathcal{A}^\rho$ transition function listed on the paper and in Section B we will present a comparison between the EFP 1.0 and the EFP 2.0 e-states.

A $m\mathcal{A}^\rho$ Transition Function Properties

A.1 Preliminary Definitions

Before starting with the demonstrations we need to introduce some terminology that will help us avoid unnecessary clutter during the proofs. In particular, let a Domain D, a $\mathsf{p} \in \mathcal{S}$. where $\mathcal{S}$ is the set of all the possibilities reachable from $D(\varphi_i)$ with a finite sequence of action instances and a group of agent $\mathcal{AG} \subseteq D(\mathcal{AG})$ be given. The operator $\mathcal{B}^{\mathsf{p}}_{\mathcal{AG}}$ captures *all the reachable possibilities for $\mathcal{AG}$ given a starting possibility* p.

Let us describe now how this operator can be used to represents the notions of i) agents' belief; ii) common knowledge; and iii) nested knowledge.

A.1.1 Agents Beliefs Representation To link the operator introduced above with the concept of belief let us start with the case where the group of agents $\mathcal{AG}$ contains only one element ag. We, therefore, use $\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}}$ to identify the set of all the possibilities that ag, starting from the possibility p, cannot distinguish.

The construction of the set identified by $\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}}$ is procedural and it is done by applying the operator $(\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})^k$, with $k \in \mathbb{N}$, until the *least fixed point* is found. The operator $(\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})^k$ is defined as follows:

$$(\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})^k = \begin{cases} \mathsf{p}(\mathsf{ag}) & \text{if } k = 0 \\ \{\mathsf{q} \mid (\exists \mathsf{u} \in (\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})^{k-1})(\mathsf{q} \in \mathsf{u}(\mathsf{ag}))\} & \text{if } k \geq 1 \end{cases}$$

Finally we can define $\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}} = \bigcup_{k=1}^{\infty} (\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})^k$. It is easy to see that this is equivalent to the set of possibilities reached by the operator $\mathbf{B}_{\mathsf{ag}}$ starting from p and, therefore, that it represents the beliefs of ag in u.

Let us note that fixed point of the operator $(\mathcal{B}^{\mathcal{S}}_{\mathcal{AG}})^k$ is reached in finite iterations. This is because:

- $(\mathcal{B}^{\mathcal{S}}_{\mathcal{AG}})^k$ is monotonic; meaning that $(\mathcal{B}^{\mathcal{S}}_{\mathcal{AG}})^k \subseteq (\mathcal{B}^{\mathcal{S}}_{\mathcal{AG}})^{k+1}$ with $k \in \mathbb{N}$ (Lemma 1); and
- the set $\mathcal{S}$ of all the possibilities reached by applying a finite action instances sequence Δ to a given possibility p s.t. $|\mathcal{B}^{\mathsf{p}}_{\mathcal{AG}}| = n$ has a finite number of elements (Proposition 1).

A.1.2 Common Knowledge Representation

Now, similarly to the single-agent case, we can define the set $\mathcal{B}^{\mathsf{p}}_{\mathcal{AG}}$. This represents the *common knowledge* of $\mathcal{AG}$ ($\mathbf{C}_{\mathcal{AG}}$) starting from p. As before we introduce the operator $(\mathcal{B}^{\mathsf{p}}_{\mathcal{AG}})^k$ of which the fixed point will result in $\mathcal{B}^{\mathsf{p}}_{\mathcal{AG}}$.

$$(\mathcal{B}^{\mathsf{p}}_{\mathcal{AG}})^k = \begin{cases} \bigcup\limits_{\mathsf{ag}\in\mathcal{AG}} \mathsf{p}(\mathsf{ag}) & \text{if } k = 0 \\ \{\mathsf{q} \mid (\exists \mathsf{u} \in (\mathcal{B}^{\mathsf{p}}_{\mathcal{AG}})^{k-1})(\mathsf{q} \in \bigcup\limits_{\mathsf{ag}\in\mathcal{AG}} \mathsf{u}(\mathsf{ag}))\} & \text{if } k \geq 1 \end{cases}$$

A.1.3 Nested Knowledge Representation

Finally, thanks to these notations, we can also express the concept of *nested knowledge* in a more compact way. Let two sets of agents $\mathcal{AG}_1 \subseteq D(\mathcal{AG}), \mathcal{AG}_2 \subseteq D(\mathcal{AG})$ be given; the set of possibilities reachable by applying $\mathbf{C}_{\mathcal{AG}_1}\mathbf{C}_{\mathcal{AG}_2}$ starting from p is:

$$\mathcal{B}^{\mathsf{p}}_{\mathcal{AG}_1,\mathcal{AG}_2} = \{\mathsf{q} \mid (\exists \mathsf{r} \in \mathcal{B}^{\mathsf{p}}_{\mathcal{AG}_1})(\mathsf{q} \in \mathcal{B}^{\mathsf{r}}_{\mathcal{AG}_2})\}$$

Let us note that, when $\mathcal{AG}_1$ or $\mathcal{AG}_2$ contains only one agent ag, the nested the operator finds the correct set of possibilities being $\mathbf{C}_{\mathsf{ag}}$ and $\mathbf{B}_{\mathsf{ag}}$ equal.

Lemma 1 (Operator $\mathcal{B}^{\mathcal{S}}_{\mathcal{AG}}$ monotony). *The operator $(\mathcal{B}^{\mathcal{S}}_{\mathcal{AG}})$ is monotonic; meaning that, for every $k \in \mathbb{N}$, $(\mathcal{B}^{\mathcal{S}}_{\mathcal{AG}})^k \subseteq (\mathcal{B}^{\mathcal{S}}_{\mathcal{AG}})^{k+1}$.*

Proof. Without losing generality let a possibility p and an agent ag be given. To demonstrate the monotonicity of $(\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})$ we start by recalling that:

$$\begin{aligned} (\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})^0 &= \{\mathsf{q} \mid (\mathsf{q} \in \mathsf{p}(\mathsf{ag}))\}; \\ (\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})^1 &= \{\mathsf{q} \mid (\exists \mathsf{u} \in (\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})^0)(\mathsf{q} \in \mathsf{u}(\mathsf{ag}))\}; \\ &\vdots \\ (\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})^k &= \{\mathsf{q} \mid (\exists \mathsf{u} \in (\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})^{k-1})(\mathsf{q} \in \mathsf{u}(\mathsf{ag}))\}. \end{aligned}$$

By construction each possibility respects the **KD45** logic (Table 1) and, therefore, some structural constraints. In particular, to comply with axioms **4** and **5**, if a possibility $\mathsf{q} \in \mathsf{p}(\mathsf{ag})$ then $\mathsf{q} \in \mathsf{q}(\mathsf{ag})$. In term of our operator, this translate into *if a possibility* $\mathsf{q} \in (\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})^{k-1}$ *then* $\mathsf{q} \in (\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})^k$.

It is easy to see that this property[3] ensures that the agent's reachability function respect introspection. That is; when an agent reaches q she has to 'know' that herself considers q possible. Thanks to this property we can now infer that each iteration of the reachability operator $(\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})^k$ contains at least $(\mathcal{B}^{\mathsf{p}}_{\mathsf{ag}})^{k-1}$ and, therefore, that the operator$(\mathcal{B}^{\mathcal{S}}_{\mathcal{AG}})$ is monotonic.

Proposition 1 (States Size Finiteness). *Given a finite action instances sequence Δ—namely a* plan*—and a starting point* i*, s.t.* $|\mathcal{B}^{\mathsf{i}}_{\mathcal{AG}}| = n$*, the set $\mathcal{S}$ of all the possibilities generated by applying Δ to* i *has a finite number of elements.*

Proof. Following Definition 1 we can determine an upper bound for the number of new possibilities generated after the application of an action instance and, moreover, of an action instance sequence. In particular from a given possibility i such that $|\mathcal{B}^{\mathsf{i}}_{\mathcal{AG}}| = n$ (where $\mathcal{AG}$ is the set of all the agents) the cardinality of the set $\mathcal{B}^{\mathsf{i}'}_{\mathcal{AG}}$ will be, at most, equal to $3n$. That is because:

[3] That translates into self-loops in the graphical state representation.

Property of $\mathcal{B}$	Axiom
$(\mathbf{B}_{\mathsf{ag}}\varphi \wedge \mathbf{B}_{\mathsf{ag}}(\varphi \Rightarrow \psi)) \Rightarrow \mathbf{B}_{\mathsf{ag}}\psi$	**K**
$\neg\mathbf{B}_{\mathsf{ag}}\bot$	**D**
$\mathbf{B}_{\mathsf{ag}}\varphi \Rightarrow \mathbf{B}_{\mathsf{ag}}\mathbf{B}_{\mathsf{ag}}\varphi$	**4**
$\neg\mathbf{B}_{\mathsf{ag}}\varphi \Rightarrow \mathbf{B}_{\mathsf{ag}}\neg\mathbf{B}_{\mathsf{ag}}\varphi$	**5**

Table 1: **KD45** axioms [2].

– when an *ontic* action is executed each possibility $\in |\mathcal{B}^{\mathsf{p}}_{\mathcal{AG}}|$ can be either updated—if reached by a fully observant agent—or kept unchanged—if reached by an oblivious agent. This means that an upper bound to the size of $\mathcal{B}^{\mathsf{p}'}_{\mathcal{AG}}$ in case of an ontic action execution is $2n$ where only the updated possibilities (n) are new elements of $\mathcal{S}$.
– The case with *sensing* and *annoucement* actions is similar

This identifies $2n$ as upper bound for the growth of a state size and for the generation of new possibilities after an action execution.Therefore given the size n of the initial state and the length of the action sequence l we can conclude that $|\mathcal{S}| \leq (n \times 2^l)$ and it is indeed finite.

A.2 $m\mathcal{A}^\rho$ Properties

In what follows we will demonstrate that the $m\mathcal{A}^\rho$ transition function respects the properties listed in the paper. Before starting the demonstrations, for the sake of readability, let us re-introduce the new transition function for $m\mathcal{A}^\rho$.

Let a domain D, its set of action instances $D(\mathcal{AI})$, and the set $\mathcal{S}$ of all the possibilities reachable from $D(\varphi_i)$ with a finite sequence of action instances be given. The transition function $\Phi : D(\mathcal{AI}) \times \mathcal{S} \to \mathcal{S} \cup \{\emptyset\}$ for $m\mathcal{A}^\rho$ relative to D is defined as follows.

Definition 1 ($m\mathcal{A}^\rho$ transition function). *Allow us to use the compact notation* $\mathsf{u}(\mathcal{F}) = \{\mathsf{f} \mid \mathsf{f} \in D(\mathcal{F}) \wedge \mathsf{u} \models \mathsf{f}\} \cup \{\neg\mathsf{f} \mid \mathsf{f} \in D(\mathcal{F}) \wedge \mathsf{u} \not\models \mathsf{f}\}$ *for the sake of readability. Let an action instance* $\mathsf{a} \in D(\mathcal{AI})$, *a possibility* $\mathsf{u} \in \mathcal{S}$ *and an agent* $\mathsf{ag} \in D(\mathcal{AG})$ *be given.*

If a *is not executable in* u, *then* $\Phi(\mathsf{a},\mathsf{u}) = \emptyset$ *otherwise* $\Phi(\mathsf{a},\mathsf{u}) = \mathsf{u}'$, *where:*

– *Let us consider the case of an* ontic *action instance* a. *We then define* u' *such that:*

$$e(\mathsf{a},\mathsf{u}) = \{\ell \mid (\mathsf{a}\ \textbf{\textit{causes}}\ \ \ell) \in D\};\ \textit{and}$$
$$\overline{e(\mathsf{a},\mathsf{u})} = \{\neg\ell \mid \ell \in e(\mathsf{a},\mathsf{u})\}\ \textit{where}\ \neg\neg\ell\ \textit{is replaced by}\ \ell.$$

$$\mathsf{u}'(\mathsf{f}) = \begin{cases} 1 & \textit{if}\ \mathsf{f} \in (\mathsf{u}(\mathcal{F}) \setminus \overline{e(\mathsf{a},\mathsf{u})}) \cup e(\mathsf{a},\mathsf{u}) \\ 0 & \textit{if}\ \neg\mathsf{f} \in (\mathsf{u}(\mathcal{F}) \setminus \overline{e(\mathsf{a},\mathsf{u})}) \cup e(\mathsf{a},\mathsf{u}) \end{cases}$$

$$\mathsf{u}'(\mathsf{ag}) = \begin{cases} \mathsf{u}(\mathsf{ag}) & \textit{if}\ \mathsf{ag} \in O_{\mathsf{a}} \\ \bigcup\limits_{\mathsf{w} \in \mathsf{u}(\mathsf{ag})} \Phi(\mathsf{a},\mathsf{w}) & \textit{if}\ \mathsf{ag} \in F_{\mathsf{a}} \end{cases}$$

– *if* a *is a* sensing *action instance, used to sense the fluent* f. *We then define* u' *such that:*

$$e(\mathsf{a},\mathsf{u}) = \{\mathsf{f} \mid (\mathsf{a}\ \textbf{\textit{senses}}\ \ \mathsf{f}) \in D \wedge \mathsf{u} \models \mathsf{f}\} \cup \{\neg\mathsf{f} \mid (\mathsf{a}\ \textbf{\textit{senses}}\ \ \mathsf{f}) \in D \wedge \mathsf{u} \not\models \mathsf{f}\}$$

$$\mathsf{u}'(\mathcal{F}) = \mathsf{u}(\mathcal{F})$$

$$\mathsf{u}'(\mathsf{ag}) = \begin{cases} \mathsf{u}(\mathsf{ag}) & \textit{if } \mathsf{ag} \in O_\mathsf{a} \\ \bigcup\limits_{\mathsf{w}\in\mathsf{u}(\mathsf{ag})} \Phi(\mathsf{a},\mathsf{w}) & \textit{if } \mathsf{ag} \in P_\mathsf{a} \\ \bigcup\limits_{\mathsf{w}\in\mathsf{u}(\mathsf{ag}):\ e(\mathsf{a},\mathsf{w})=e(\mathsf{a},\mathsf{u})} \Phi(\mathsf{a},\mathsf{w}) & \textit{if } \mathsf{ag} \in F_\mathsf{a} \end{cases}$$

– *if* a *is an* announcement *action instance of the fluent formula* ϕ*. We then define* u' *such that:*

$$e(\mathsf{a},\mathsf{u}) = \begin{cases} 0 & \textit{if } \mathsf{u} \models \phi \\ 1 & \textit{if } \mathsf{u} \models \neg\phi \end{cases}$$

$$\mathsf{u}'(\mathcal{F}) = \mathsf{u}(\mathcal{F})$$

$$\mathsf{u}'(\mathsf{ag}) = \begin{cases} \mathsf{u}(\mathsf{ag}) & \textit{if } \mathsf{ag} \in O_\mathsf{a} \\ \bigcup\limits_{\mathsf{w}\in\mathsf{u}(\mathsf{ag})} \Phi(\mathsf{a},\mathsf{w}) & \textit{if } \mathsf{ag} \in P_\mathsf{a} \\ \bigcup\limits_{\mathsf{w}\in\mathsf{u}(\mathsf{ag}):\ e(\mathsf{a},\mathsf{w})=e(\mathsf{a},\mathsf{u})} \Phi(\mathsf{a},\mathsf{w}) & \textit{if } \mathsf{ag} \in F_\mathsf{a} \end{cases}$$

A.3 Properties of $m\mathcal{A}^\rho$

We will now proceed to demonstrate the properties to prove that in $m\mathcal{A}^\rho$ holds what follows.

– If an agent is fully aware of the execution of an action instance then her beliefs will be updated with the effects of such action execution;
– An agent who is only partially aware of the action occurrence will believe that the agents who are fully aware of the action occurrence are certain about the action's effects; and
– An agent who is oblivious of the action occurrence will also be ignorant about its effects.

In the following proofs we will use p' instead of $\Phi(\mathsf{a},\mathsf{p})$ to avoid unnecessary clutter when possible.

Proposition 2 (Ontic Action Properties). *Assume that* a *is an* ontic action instance *executable in* u *s.t.* a ***causes*** l *if* ψ *belongs to* D*. In* $m\mathcal{A}^\rho$ *it holds that:*

1. *for every agent* $\mathsf{x} \in F_\mathsf{a}$*, if* $\mathsf{u} \models \mathbf{B}_\mathsf{x}\psi$ *then* $\mathsf{u}' \models \mathbf{B}_\mathsf{x} l$*;*
2. *for every agent* $\mathsf{y} \in O_\mathsf{a}$ *and a belief formula* φ*,* $\mathsf{u}' \models \mathbf{B}_\mathsf{y}\varphi$ *iff* $\mathsf{u} \models \mathbf{B}_\mathsf{y}\varphi$*; and*
3. *for every pair of agents* $\mathsf{x} \in F_\mathsf{a}$ *and* $\mathsf{y} \in O_\mathsf{a}$ *and a belief formula* φ*, if* $\mathsf{u} \models \mathbf{B}_\mathsf{x}\mathbf{B}_\mathsf{y}\varphi$ *then* $\mathsf{u}' \models \mathbf{B}_\mathsf{x}\mathbf{B}_\mathsf{y}\varphi$*.*

Proof. We will prove each point separately:

1. Assuming the action a is executable in u we have that $\mathsf{u} \models \psi$. This means that:
   – If $\mathsf{u} \models \mathbf{B}_\mathsf{x}\psi$ we have that $\forall \mathsf{p} \in \mathcal{B}^\mathsf{u}_\mathsf{x}\ \mathsf{p} \models \psi$; this is because, as said in Section A.1.1, $\mathcal{B}^\mathsf{u}_\mathsf{x}$ represents the set of possibilities reachable by $\mathbf{B}_\mathsf{x}$ starting from u.
   – In particular we are interested in the set of possibilities reachable by $\mathbf{B}_\mathsf{x}$ starting from u', *i.e.*, $\mathcal{B}^{\mathsf{u}'}_\mathsf{x} = \{\mathsf{p}' \mid (\exists \mathsf{p} \in \mathcal{B}^\mathsf{u}_\mathsf{x})(\mathsf{p}' = \Phi(\mathsf{p},\mathsf{a}))\}$.

- Following Definition 1, we also know that—being $\mathsf{x} \in F_{\mathsf{a}}$—if $\ell = \mathsf{f}$[4] then $e(\mathsf{a}, \mathsf{u}) = \{\mathsf{f}\}$ and therefore $\mathsf{p}'(\mathsf{f}) = 1 \ \forall \mathsf{p}' \in \mathcal{B}^{\mathsf{u}'}_{\mathsf{x}}$.
- From this last step we can conclude that every element of $\mathcal{B}^{\mathsf{u}'}_{\mathsf{ag}}$ entails f.
- As said previously $\mathcal{B}^{\mathsf{u}'}_{\mathsf{x}}$ represents $\mathbf{B}_{\mathsf{x}}$ starting from u'.
- It is easy to see that, if every element in $\mathcal{B}^{\mathsf{u}'}_{\mathsf{x}}$ entails f, then $\mathsf{u}' \models \mathbf{B}_{\mathsf{x}} f$.

2. As in the previous point we assume the action a is executable in u and this means that:
   - If $\mathsf{u} \models \mathbf{B}_{\mathsf{y}} \varphi$ we have that every $\mathsf{p} \in \mathcal{B}^{\mathsf{u}}_{\mathsf{y}}$ entails φ.
   - Given that, from Definition 1, when $\mathsf{y} \in O_{\mathsf{a}}$ for each possibility $\mathsf{p} \in \mathcal{B}^{\mathsf{u}}_{\mathsf{y}}$ $\mathsf{p}(\mathsf{y}) = \mathsf{p}'(\mathsf{y})$ it is easy to see that $\mathcal{B}^{\mathsf{u}}_{\mathsf{y}} \equiv \mathcal{B}^{\mathsf{u}'}_{\mathsf{y}}$.
   - Given that the two sets of possibilities are the same it means that the reachability functions that they represent are the same.
   - Being the two functions the same it means that $\forall \varphi \in D$ $\mathsf{u} \models \mathbf{B}_{\mathsf{y}} \varphi$ *iff* $\mathsf{u}' \models \mathbf{B}_{\mathsf{y}} \varphi$.

3. Again we assume the executability of the action a and we consider $\mathsf{x} \in F_{\mathsf{a}}$ and $\mathsf{y} \in O_{\mathsf{a}}$:
   - Being $\mathsf{y} \in O_{\mathsf{a}}$, from Definition 1, we know that $\mathsf{p}(\mathsf{y}) = \mathsf{p}'(\mathsf{y})$ such that $\mathsf{p} \in \mathcal{B}^{\mathsf{u}}_{\mathsf{x}}$ and p' is its updated version $\in \mathcal{B}^{\mathsf{u}'}_{\mathsf{x}}$.
   - This means that for every element in $\mathcal{B}^{\mathsf{u}}_{\mathsf{x}}$ we have an updated version that has the same reachability function for the agent y.
   - Then it is easy to see that $\mathcal{B}^{\mathsf{u}}_{\mathsf{x},\mathsf{y}} \equiv \mathcal{B}^{\mathsf{u}'}_{\mathsf{x},\mathsf{y}}$ and therefore that these two sets contain the same possibilities.
   - As already said in Point 2 when two sets of possibilities are the same they entail the same formulae.
   - Therefore we can conclude that if $\mathsf{u} \models \mathbf{B}_{\mathsf{x}} \mathbf{B}_{\mathsf{y}} \varphi$ then $\mathsf{u}' \models \mathbf{B}_{\mathsf{x}} \mathbf{B}_{\mathsf{y}} \varphi$

Proposition 3 (Sensing Action Properties). *Assume that* a *is a* sensing action instance *and* D *contains the statement* a ***determines*** f. *In* $m\mathcal{A}^{\rho}$ *it holds that:*

1. *if* $\mathsf{u} \models \mathsf{f}$ *then* $\mathsf{u}' \models \mathbf{C}_{F_{\mathsf{a}}} \mathsf{f}$;
2. *if* $\mathsf{u} \models \neg \mathsf{f}$ *then* $\mathsf{u}' \models \mathbf{C}_{F_{\mathsf{a}}} \neg \mathsf{f}$;
3. $\mathsf{u}' \models \mathbf{C}_{P_{\mathsf{a}}}(\mathbf{C}_{F_{\mathsf{a}}} \mathsf{f} \vee \mathbf{C}_{F_{\mathsf{a}}} \neg \mathsf{f})$;
4. $\mathsf{u}' \models \mathbf{C}_{F_{\mathsf{a}}}(\mathbf{C}_{P_{\mathsf{a}}}(\mathbf{C}_{F_{\mathsf{a}}} \mathsf{f} \vee \mathbf{C}_{F_{\mathsf{a}}} \neg \mathsf{f}))$;

[4] The case where a **causes** $\neg \mathsf{f}$ is similar and, therefore, is omitted here

5. *for every agent* $\mathsf{y} \in O_{\mathsf{a}}$ *and a belief formula* φ, $\mathsf{u}' \models \mathbf{B}_{\mathsf{y}}\varphi$ *iff* $\mathsf{u} \models \mathbf{B}_{\mathsf{y}}\varphi$; *and*
6. *for every pair of agents* $\mathsf{x} \in F_{\mathsf{a}}$ *and* $\mathsf{y} \in O_{\mathsf{a}}$ *and a belief formula* φ, *if* $\mathsf{u} \models \mathbf{B}_{\mathsf{x}}\mathbf{B}_{\mathsf{y}}\varphi$ *then* $\mathsf{u}' \models \mathbf{B}_{\mathsf{x}}\mathbf{B}_{\mathsf{y}}\varphi$.

Proof. Let us demonstrate each point separately:

1. In the following we demonstrate Point 1. Being the demonstration for Point 2 similar we will omit it for the sake of readability.

   – First of all we identify the set of all the possibilities reached by the *fully observant* agents in u as $\mathcal{B}^{\mathsf{u}}_{F_{\mathsf{a}}}$ and we remind that, as shown in Section A.1.2, this set corresponds to the possibilities reached by $\mathbf{C}_{F_{\mathsf{a}}}$;

   – We recall that, by hypothesis, $\mathsf{u} \models \mathsf{f}$ and therefore $e(\mathsf{a}, \mathsf{u}) = \{\mathsf{f}\}$.

   – We then calculate $\mathcal{B}^{\mathsf{u}'}_{F_{\mathsf{a}}}$ that, following Definition 1, contains only possibilities p' s.t $\mathsf{p}'(\mathsf{f}) = 1$.

   – This means that $\forall \mathsf{p}' \in \mathcal{B}^{\mathsf{u}'}_{F_{\mathsf{a}}}$ we have that $\mathsf{p}' \models \mathsf{f}$.

   – As shown in Point 1 of Theorem 2 given that this set contains only the possibilities that entail f we can derive that $\mathcal{B}^{\mathsf{u}'}_{F_{\mathsf{a}}} \models \mathsf{f}$.

   – Finally, as the set $\mathbf{C}_{F_{\mathsf{a}}} \equiv \mathcal{B}^{\mathsf{u}'}_{F_{\mathsf{a}}}$, we have that $\mathbf{C}_{F_{\mathsf{a}}} \models \mathsf{f}$.

2. The proof of this point is similar to the one presented in Point 1 and it is omitted for the sake of readability.

3. Once again we identify the set of the possibilities reachable by *partial observants* agent with $\mathcal{B}^{\mathsf{u}}_{P_{\mathsf{a}}}$. We also remind that this set is equal to $\mathbf{C}_{P_{\mathsf{a}}}$ in u.

   – Now to calculate $\mathcal{B}^{\mathsf{u}'}_{P_{\mathsf{a}}}$, following Definition 1, we apply "$\Phi(\mathsf{a}, \mathsf{u})$" to every element of $\mathcal{B}^{\mathsf{u}}_{P_{\mathsf{a}}}$.

   – To simplify the demonstration let us redefine the partially observant agents' belief update for epistemic actions in the following way:

$$\mathsf{u}'(\mathsf{ag}) = \begin{cases} \bigcup\limits_{\mathsf{w} \in \mathsf{u}(\mathsf{ag})} \Phi(\mathsf{a}, \mathsf{w}) & \text{if } \mathsf{ag} \in \mathcal{AG}, \mathsf{ag} \in P_{\mathsf{a}} \text{ and } e(\mathsf{a}, \mathsf{u}) = e(\mathsf{a}, \mathsf{w}) \\ \bigcup\limits_{\mathsf{w} \in \mathsf{u}(\mathsf{ag})} \Phi(\mathsf{a}, \mathsf{w}) & \text{if } \mathsf{ag} \in \mathcal{AG}, \mathsf{ag} \in P_{\mathsf{a}} \text{ and } e(\mathsf{a}, \mathsf{u}) \neq e(\mathsf{a}, \mathsf{w}) \end{cases} \qquad \text{Where } \mathsf{ag} \in P_{\mathsf{a}}$$

   – It is easy to identify two disjunct subsets $\mathcal{B}^{1}_{P_{\mathsf{a}}}$ and $\mathcal{B}^{2}_{P_{\mathsf{a}}}$ of $\mathcal{B}^{\mathsf{u}'}_{P_{\mathsf{a}}}$ that contains only possibility such that:
     - $\mathcal{B}^{1}_{P_{\mathsf{a}}} \models e(\mathsf{a}, \mathsf{u})$;
     - $\mathcal{B}^{2}_{P_{\mathsf{a}}} \not\models e(\mathsf{a}, \mathsf{u})$;
     - $(\mathcal{B}^{1}_{P_{\mathsf{a}}} \cup \mathcal{B}^{2}_{P_{\mathsf{a}}}) \equiv \mathcal{B}^{\mathsf{u}'}_{P_{\mathsf{a}}}$; and

- $(\mathcal{B}^1_{P_\mathsf{a}} \cap \mathcal{B}^2_{P_\mathsf{a}}) \equiv \emptyset$.

– From these two sets we can now construct the sets $\mathcal{B}^1_{P_\mathsf{a},F_\mathsf{a}}$ and $\mathcal{B}^2_{P_\mathsf{a},F_\mathsf{a}}$ that are simply the set of possibilities reachable from the *fully observant* agents starting from $\mathcal{B}^1_{P_\mathsf{a}}$ and $\mathcal{B}^2_{P_\mathsf{a}}$ respectively.

– Given that the set $\mathcal{B}^1_{P_\mathsf{a},F_\mathsf{a}}$ resulted from the application of the transition function from the point of view of fully observant agents, we know from Point 1 of Theorem 2 that for $\forall \mathsf{p} \in \mathcal{B}^1_{P_\mathsf{a},F_\mathsf{a}}, \mathsf{p} \models \mathsf{f}$.

– This imply that $\mathcal{B}^1_{P_\mathsf{a},F_\mathsf{a}}$ reaches only possibilities where the interpretation of f is true and similarly in $\mathcal{B}^2_{P_\mathsf{a},F_\mathsf{a}}$ only possibilities where the interpretation of f is false.

– This means that $\mathcal{B}^1_{P_\mathsf{a},F_\mathsf{a}} \models \mathsf{f}$ and $\mathcal{B}^2_{P_\mathsf{a},F_\mathsf{a}} \models \neg\mathsf{f}$.

– It is easy to see then that $\mathcal{B}^1_{P_\mathsf{a}} \models \mathbf{C}_{F_\mathsf{a}}\mathsf{f}$ being $\mathcal{B}^1_{P_\mathsf{a},F_\mathsf{a}} = \{\mathsf{p} \mid \mathsf{p} \in \bigcup_{\mathsf{q}\in\mathcal{B}^1_{P_\mathsf{a}}} \mathsf{q}(F_\mathsf{a})\}$ (and similarly $\mathcal{B}^2_{P_\mathsf{a}} \models \mathbf{C}_{F_\mathsf{a}}\neg\mathsf{f}$).

– Finally being $\mathcal{B}^{\mathsf{u}'}_{P_\mathsf{a}} = \mathcal{B}^1_{P_\mathsf{a}} \cup \mathcal{B}^2_{P_\mathsf{a}}$ we can conclude that $\mathcal{B}^{\mathsf{u}'}_{P_\mathsf{a}} \models \mathbf{C}_{F_\mathsf{a}}\mathsf{f} \vee \mathbf{C}_{F_\mathsf{a}}\neg\mathsf{f}$[5] and therefore $\mathsf{u}' \models \mathbf{C}_{P_\mathsf{a}}(\mathbf{C}_{F_\mathsf{a}}\mathsf{f} \vee \mathbf{C}_{F_\mathsf{a}}\neg\mathsf{f})$.

4. To prove this point we will make use of the properties demonstrated in previous Points.

– As said in the Section A.1.3, we know that $\mathcal{B}^{\mathsf{u}}_{F_\mathsf{a},P_\mathsf{a}}$ corresponds with the set of possibilities identified by $\mathbf{C}_{F_\mathsf{a}}\mathbf{C}_{P_\mathsf{a}}$ and it is also equal to $\{\mathsf{p} \mid (\exists \mathsf{q} \in \mathcal{B}^{\mathsf{u}}_{P_\mathsf{a}})(\mathsf{p} \in \bigcup_{\mathsf{ag}\in F_\mathsf{a}} \mathsf{q}(\mathsf{ag}))\}$.

– Now to calculate $\mathcal{B}^{\mathsf{u}'}_{F_\mathsf{a}}$ we apply Definition 1 to every element of $\mathcal{B}^{\mathsf{u}}_{F_\mathsf{a}}$. This means that $\mathcal{B}^{\mathsf{u}'}_{F_\mathsf{a}} = \{\mathsf{p}' \mid (\exists \mathsf{p} \in \mathcal{B}^{\mathsf{u}}_{F_\mathsf{a}})(\mathsf{p}' = \Phi(\mathsf{a}, \mathsf{p}))\}$.

– We then want to calculate the set $\{\mathsf{p}' \mid (\exists \mathsf{q}' \in \mathcal{B}^{\mathsf{u}'}_{F_\mathsf{a}})(\mathsf{p}' \in \bigcup_{\mathsf{ag}\in P_\mathsf{a}} \mathsf{q}'(\mathsf{ag}))\}$.

– To calculate the "point of view" of the partially observants w.r.t. the fully observants we apply Definition 1 to all the elements of $\{\mathsf{p} \mid (\exists \mathsf{q}' \in \mathcal{B}^{\mathsf{u}'}_{F_\mathsf{a}})(\mathsf{p} \in \mathcal{B}^{\mathsf{q}}_{P_\mathsf{a}})\}$.

– It is easy to see that the resulting set is $\{\mathsf{p}' \mid (\exists \mathsf{q}' \in \mathcal{B}^{\mathsf{u}'}_{F_\mathsf{a}})(\mathsf{p}' \in \bigcup_{\mathsf{ag}\in P_\mathsf{a}} \mathsf{q}'(\mathsf{ag}))\} \equiv \mathcal{B}^{\mathsf{u}'}_{F_\mathsf{a},P_\mathsf{a}}$.

– We showed in the previous point that given the set of possibilities resulted by applying the transition function entails $\mathbf{C}_{F_\mathsf{a}}\mathsf{f} \vee \mathbf{C}_{F_\mathsf{a}}\neg\mathsf{f}$.

– This means that $\mathcal{B}^{\mathsf{u}'}_{F_\mathsf{a},P_\mathsf{a}} \models (\mathbf{C}_{F_\mathsf{a}}\mathsf{f} \vee \mathbf{C}_{F_\mathsf{a}}\neg\mathsf{f})$ and therefore, following what said in Section A.1.3, $\mathsf{u}' \models \mathbf{C}_{F_\mathsf{a}}(\mathbf{C}_{P_\mathsf{a}}(\mathbf{C}_{F_\mathsf{a}}\mathsf{f} \vee \mathbf{C}_{F_\mathsf{a}}\neg\mathsf{f}))$.

[5] The two sets are completely disjuntive as one only contains possibilities that entails f while the other only possibilities that do not. This means that that does not exist any fully-observant-edge between possibilities that belongs in two different sets.

5–6 The proofs for the fifth and sixth points are similar to the ones presented in Point 2 and Point 3 of Theorem 2 respectively and is therefore omitted.

Proposition 4 (Announcement Action Properties). *Assume that* a *is a* announcement action instance *and* D *contains the statement* a ***announces*** φ*. If* $\mathsf{u} \models \phi$ *it holds that:*

1. $\mathsf{u}' \models \mathbf{C}_{F_\mathsf{a}}\phi$;
2. $\mathsf{u}' \models \mathbf{C}_{P_\mathsf{a}}(\mathbf{C}_{F_\mathsf{a}}\phi \vee \mathbf{C}_{F_\mathsf{a}}\neg\phi)$;
3. $\mathsf{u}' \models \mathbf{C}_{F_\mathsf{a}}(\mathbf{C}_{P_\mathsf{a}}(\mathbf{C}_{F_\mathsf{a}}\phi \vee \mathbf{C}_{F_\mathsf{a}}\neg\phi))$;
4. *for every agent* $\mathsf{y} \in O_\mathsf{a}$ *and a belief formula* φ, $\mathsf{u}' \models \mathbf{B}_\mathsf{y}\varphi$ *iff* $\mathsf{u} \models \mathbf{B}_\mathsf{y}\varphi$*; and*
5. *for every pair of agents* $\mathsf{x} \in F_\mathsf{a}$ *and* $\mathsf{y} \in O_\mathsf{a}$ *and a belief formula* φ*, if* $\mathsf{u} \models \mathbf{B}_\mathsf{x}\mathbf{B}_\mathsf{y}\varphi$ *then* $\mathsf{u}' \models \mathbf{B}_\mathsf{x}\mathbf{B}_\mathsf{y}\varphi$.

Proof. The demonstration of this proposition follows of Proposition 3 and is therefore omitted for the sake of the readability.

B e-States Comparison

In this Section we will show some comparison of e-states size between EFP 1.0 and P-MAR. In particular the goal state, since it is reached after a *sensing* action execution, is the one that differs in the size.

We will use an example from the *Coin in the Box* domain that is used in [1], that is Example 10 of [1], to show their transition function. Firstly we will introduce the example and the we will show a side-to-side comparison of the e-states generated during the solving process.

Both Kripke structures and possibilities will be presented as labeled graph. Moreover, each e-state representation will be provided with a table that describes the information of each node.

Before we start let us rapidly introduce the example.

Example 1. The initial state is defined by the conditions:

1. **intially** $\mathbf{C}_{\mathsf{A,B,C}}(\mathtt{key}(\mathsf{A}))$
2. **intially** $\mathbf{C}_{\mathsf{A,B,C}}(\neg\mathtt{key}(\mathsf{B}))$
3. **intially** $\mathbf{C}_{\mathsf{A,B,C}}(\neg\mathtt{key}(\mathsf{C}))$
4. **intially** $\mathbf{C}_{\mathsf{A,B,C}}(\neg\mathtt{opened})$
5. **intially** $\mathbf{C}_{\mathsf{A,B,C}}(\neg\mathbf{B}_{\mathsf{ag}}\mathtt{tails} \wedge \neg\mathbf{B}_{\mathsf{ag}}\neg\mathtt{tails})$ for $\mathsf{ag} \in \{\mathsf{A,B,C}\}$
6. **intially** $\mathbf{C}_{\mathsf{A,B,C}}(\mathtt{looking}(\mathsf{ag}))$ for $\mathsf{ag} \in \{\mathsf{A,B,C}\}$
7. **intially** $\mathtt{tails}$

The goal is expressed trough the following formulae:

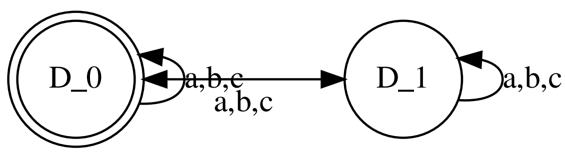
$$\mathbf{B}_\mathsf{A}\neg\mathtt{heads} \wedge \mathbf{B}_\mathsf{A}(\mathbf{B}_\mathsf{B}(\mathbf{B}_\mathsf{A}\mathtt{heads} \vee \mathbf{B}_\mathsf{A}\neg\mathtt{heads}))$$
$$\mathbf{B}_\mathsf{B}(\mathbf{B}_\mathsf{A}\mathtt{heads} \vee \mathbf{B}_\mathsf{A}\neg\mathtt{heads}) \wedge (\neg\mathbf{B}_\mathsf{B}\mathtt{heads} \wedge \neg\mathbf{B}_\mathsf{B}\neg\mathtt{heads})$$
$$\mathbf{B}_\mathsf{C}[\bigwedge_{\mathsf{ag}\in\{\mathsf{A,B,C}\}}(\neg\mathbf{B}_{\mathsf{ag}}\mathtt{heads} \wedge \neg\mathbf{B}_{\mathsf{ag}}\neg\mathtt{heads})]$$

Finally the observability relations of each action instance in Δ_c is expressed in the following Table:

	distract(C)⟨A⟩	open⟨A⟩	peek⟨A⟩
F_D	A, B, C	A, B	A
P_D	-	-	B
O_D	-	C	C

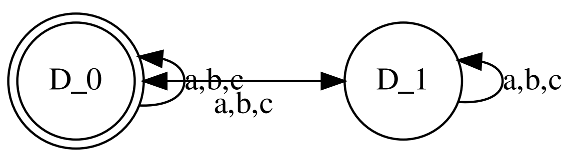
Table 2: Observability relations of the actions instances in Δ_c.

Given the initial conditions we have that the action instances sequence $\Delta_c = \texttt{distract(C)}\langle\texttt{A}\rangle; \texttt{open}\langle\texttt{A}\rangle; \texttt{peek}\langle\texttt{A}\rangle$ leads to the desired goal. In what follows this we want to give a graphical explanation of both the transition functions and state-size defined by the two solvers. The e-states are automatically generated by the planners.



D_0	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, tail
D_1	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, -tail

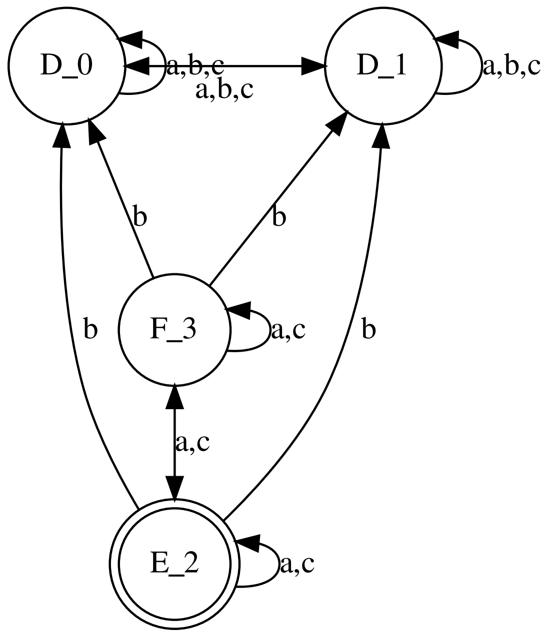
(a) The initial e-state in EFP 1.0.



D_0	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, tail
D_1	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, -tail

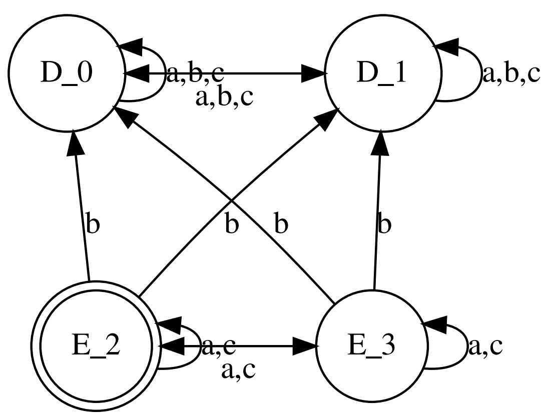
(b) The initial e-state in P-MAR.

Figure 1: The initial state.



D_0	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, tail
D_1	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, -tail
E_2	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, -opened, tail
F_3	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, -opened, -tail

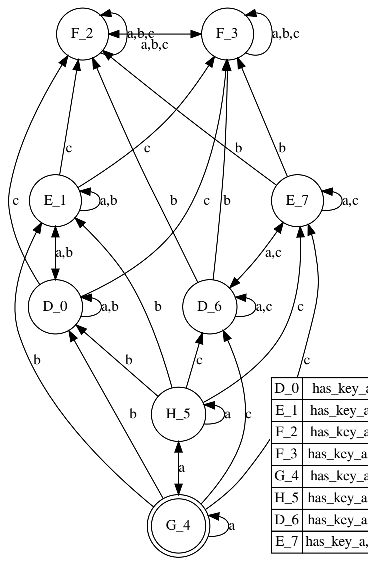
(a) The e-state obtained after the execution of `distract`(C)⟨A⟩ in EFP 1.0.



D_0	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, tail
D_1	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, -tail
E_2	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, -opened, tail
E_3	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, -opened, -tail

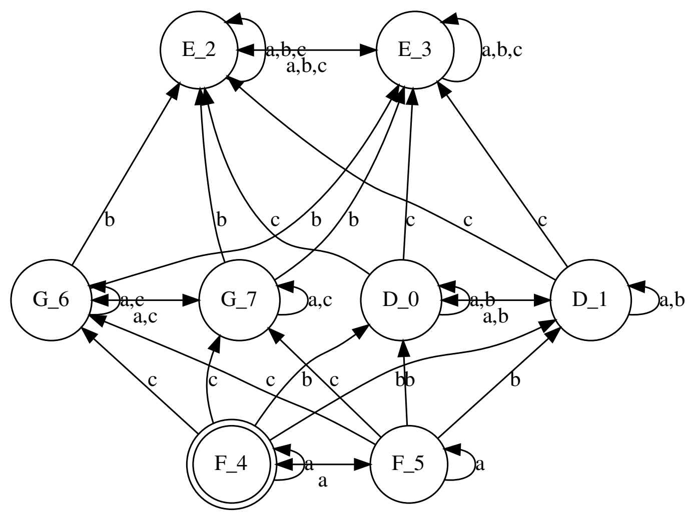
(b) The e-state obtained after the execution of `distract`(C)⟨A⟩ in P-MAR.

Figure 2: The initial state.



D_0	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, opened, tail
E_1	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, opened, -tail
F_2	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, tail
F_3	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, -tail
G_4	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, opened, tail
H_5	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, opened, -tail
D_6	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, -opened, tail
E_7	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, -opened, -tail

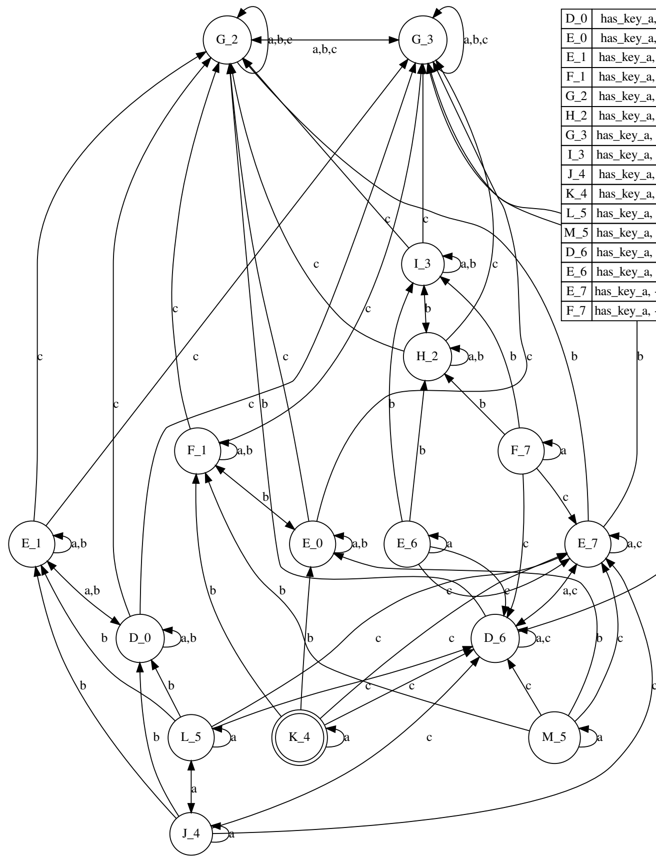
(a) TThe e-state obtained after the execution of open⟨A⟩ in EFP 1.0.



D_0	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, opened, tail
D_1	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, opened, -tail
E_2	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, tail
E_3	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, -tail
F_4	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, opened, tail
F_5	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, opened, -tail
G_6	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, -opened, tail
G_7	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, -opened, -tail

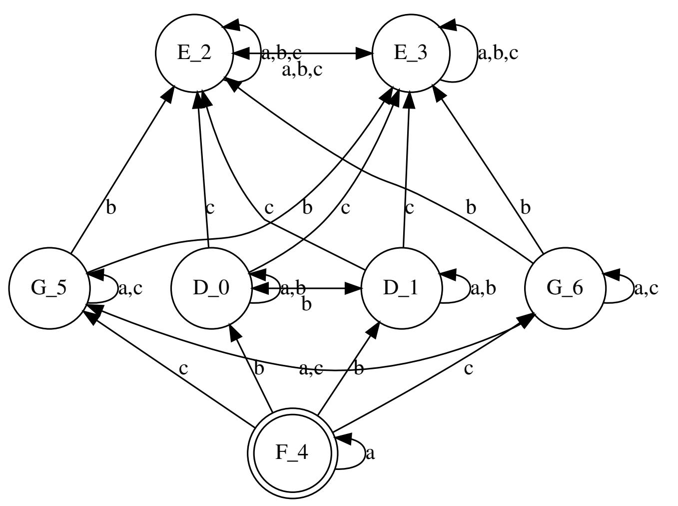
(b) The e-state obtained after the execution of open⟨A⟩ in P-MAR.

Figure 3: The initial state.



D_0	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, opened, tail
E_0	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, opened, tail
E_1	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, opened, -tail
F_1	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, opened, -tail
G_2	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, tail
H_2	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, tail
G_3	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, -tail
I_3	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, -tail
J_4	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, opened, tail
K_4	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, opened, tail
L_5	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, opened, -tail
M_5	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, opened, -tail
D_6	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, -opened, tail
E_6	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, -opened, tail
E_7	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, -opened, -tail
F_7	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, -opened, -tail

(a) The e-state obtained after the execution of peek⟨A⟩ in EFP 1.0.



D_0	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, opened, tail
D_1	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, opened, -tail
E_2	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, tail
E_3	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, looking_c, -opened, -tail
F_4	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, opened, tail
G_5	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, -opened, tail
G_6	has_key_a, -has_key_b, -has_key_c, looking_a, looking_b, -looking_c, -opened, -tail

(b) The e-state obtained after the execution of peek⟨A⟩ in P-MAR.

Figure 4: The initial state.

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1. Baral, C., Gelfond, G., Pontelli, E., Son, T.C.: An action language for multi-agent domains: Foundations. CoRR **abs/1511.01960** (2015), http://arxiv.org/abs/1511.01960
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