University of Udine
Department of Mathematics, Computer Science and Physics

MODELLING MULTI-AGENT EPISTEMIC PLANNING IN ASP

36th International Conference on Logic Programming

Alessandro Burigana, Francesco Fabiano, Agostino Dovier and Enrico Pontelli

August 31, 2022

Overview

Chapter 1

Multi-Agent Epistemic Planning

Introduction

Epistemic Reasoning

Reasoning not only about agents' perception of the world but also about agents' knowledge and/or beliefs of her and others' beliefs.

Introduction

Epistemic Reasoning

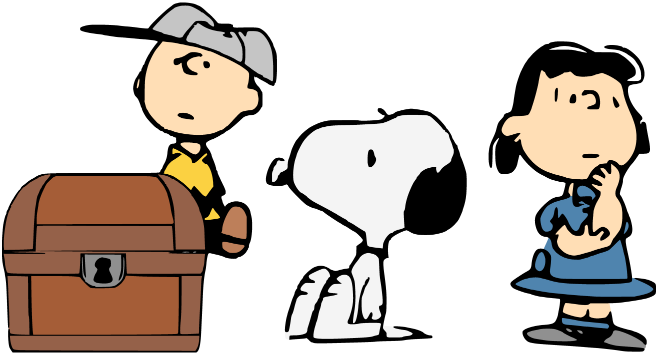
Reasoning not only about agents' perception of the world but also about agents' knowledge and/or beliefs of her and others' beliefs.

Multi-agent Epistemic Planning Problem **bolander2011epistemic**

Finding plans where the goals can refer to:

- the state of the world
- the knowledge and/or the beliefs of the agents

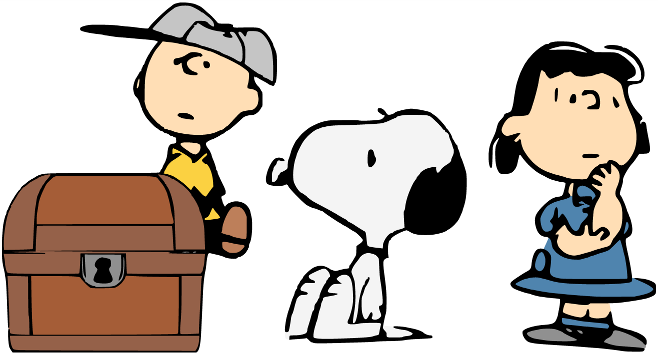
An Example



An Example

Initial State

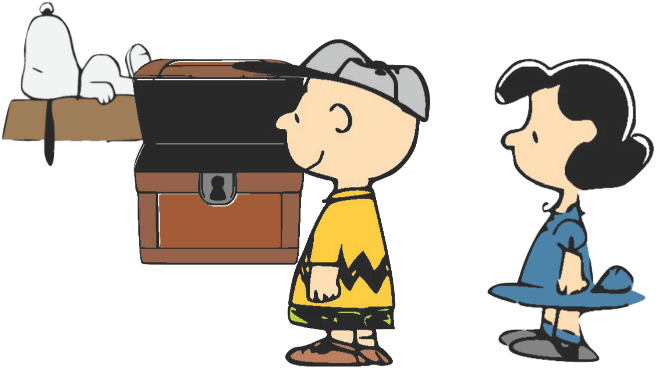
- Snoopy and Charlie are looking while Lucy is ¬looking
- No one knows the coin position.



An Example

Goal State

- Charlie knows the coin position
- Lucy knows that Charlie knows the coin position
- Snoopy does not know anything about the plan execution



Notations

Given a set of agents

Belief formulae where ag $\in$, $\alpha \subseteq$

We use the operators B_{ag} and C_{α} to model the beliefs and the common knowledge of the agents.

Notations

Given a set of agents

Belief formulae

where ag ∈ , $\alpha \subseteq$

We use the operators B_{ag} and C_α to model the beliefs and the common knowledge of the agents.

Properties of B_{ag}

KD45 and **S5** Axioms

Given the fluent formulae ϕ, ψ and the worlds $\mathtt{i}$, $\mathtt{j}$

D	$\neg\mathcal{R}_\mathtt{i}\bot$	$\mathcal{B}$ $\mathcal{K}$
K	$(\mathcal{R}_\mathtt{i}\varphi \wedge \mathcal{R}_\mathtt{i}(\varphi \implies \psi)) \implies \mathcal{R}_\mathtt{i}\psi$	$\mathcal{B}$ $\mathcal{K}$
T	$\mathcal{R}_\mathtt{i}\varphi \implies \varphi$	$\mathcal{K}$
4	$\mathcal{R}_\mathtt{i}\varphi \implies \mathcal{R}_\mathtt{i}\mathcal{R}_\mathtt{i}\varphi$	$\mathcal{B}$ $\mathcal{K}$
5	$\neg\mathcal{R}_\mathtt{i}\varphi \implies \mathcal{R}_\mathtt{i}\neg\mathcal{R}_\mathtt{i}\varphi$	$\mathcal{B}$ $\mathcal{K}$

Chapter 2

Possibilities

Overview

- Introduced by Gerbrandy and Groeneveld **Gerbrandy1997**
- Used to represent multi-agent information change
- Based on non-well-founded sets

Formal Definition

Possibility **Gerbrandy1997**

Let be a set of agents and $\mathcal{F}$ a set of propositional variables:

- A possibility u is a function that assigns to each propositional variable l $\in \mathcal{F}$ a truth value u(l) $\in \{0, 1\}$ and to each agent $ag \in$ a set of possibilities u(ag) $= \sigma$ (information state).

Intuitively ...

- The possibility u is a possible interpretation of the world and of the agents' beliefs
- u(l) specifies the truth value of the literal l
- u(ag) is the set of all the interpretations the agent ag considers possible in u
- Representable with graphs: we will use graph terminology

Chapter 3

The action language $m\mathcal{A}^{\rho}$

Action types

We introduced the action language $m\mathcal{A}^\rho$ in **Fab20**

- Used to describe MEP problems
- Uses possibilities as states
- Actions preconditions: belief formulae

Action types

We introduced the action language $m\mathcal{A}^\rho$ in **Fab20**

- Used to describe MEP problems
- Uses possibilities as states
- Actions preconditions: belief formulae

Three types of actions:

- Ontic: modifies some fluents of the world

  Charlie *opens* the box

Action types

We introduced the action language $m\mathcal{A}^\rho$ in **Fab20**

- Used to describe MEP problems
- Uses possibilities as states
- Actions preconditions: belief formulae

Three types of actions:

- Ontic: modifies some fluents of the world

  Charlie *opens* the box
- Sensing: senses the true value of a fluent

  Charlie *peeks* inside the box

Action types

We introduced the action language $m\mathcal{A}^\rho$ in **Fab20**

- Used to describe MEP problems
- Uses possibilities as states
- Actions preconditions: belief formulae

Three types of actions:

- Ontic: modifies some fluents of the world

  Charlie *opens* the box
- Sensing: senses the true value of a fluent

  Charlie *peeks* inside the box
- Announcement: announces the fluent to other agents

  Charlie *announces* the coin position

Observability Relations

An execution of an action might change or not an agents' belief accordingly to her degree of awareness

Action type	Full observers	Partial Observers	Oblivious
Ontic	✓		✓
Sensing	✓	✓	✓
Announcement	✓	✓	✓

Chapter 4

PLATO

Overview

PLATO, e**P**istemic mu**L**ti-agent **A**nswer se**T** programming s**O**lver:

- Declarative encoding in `ASP` of MEP
- Based on the language $m\mathcal{A}^\rho$
- Main components: *initial state generation*, *entailment*, *transition function*
- Exploits *clingo*'s multi-shot capabilities **Geb19**
- Formal proof of correctness

Encoding possibilities

Let u be a possibility.

ASP encoding: possibilities

We encode u with the atom possible_world(T_u, R_u, P_u), where:

- T_u tells us when u was created
- R_u is the *repetition* of u
- P_u is the numerical index of u

Encoding possibilities

Let u be a possibility.

ASP encoding: possibilities

We encode u with the atom possible_world(T_u, R_u, P_u), where:

- T_u tells us when u was created
- R_u is the *repetition* of u
- P_u is the numerical index of u

ASP encoding: pointed possibility

If u is the possibility that represents the *real configuration* of the world, we encode it with the atom pointed(T_u, R_u, P_u).

When the context is clear we will use *only* P_u instead of (T_u, R_u, P_u).

Encoding possibilities

Let u, v be two possibilities, let ag be an agent and let F be a fluent.

ASP encoding: information states

We encode $v \in u(ag)$ with the atom $\texttt{believes}(P_u, P_v, ag)$.

Encoding possibilities

Let u, v be two possibilities, let ag be an agent and let F be a fluent.

ASP encoding: information states

We encode $v \in u(ag)$ with the atom $\mathtt{believes}(P_u, P_v, ag)$.

ASP encoding: interpretations

We encode $u(F) = 1$ with the atom $\mathtt{holds}(P_u, F)$.

Entailment

Given a possibility P and a belief formula F.

Entailment

Given a possibility P and a belief formula F.

```
entails (P,          F)  :- holds(P, F), fluent(F).
entails (P,     neg(F))  :- not entails(P, F).
entails (P, and(F1, F2)) :- entails(P, F1), entails(P, F2).
entails (P,  or(F1, F2)) :- entails(P, F1).
entails (P,  or(F1, F2)) :- entails(P, F2).
```

Entailment

Given a possibility P and a belief formula F.

```
entails     (P,          F)   :- holds(P,F), fluent(F).
entails     (P,     neg(F))   :- not entails(P,F).
entails     (P, and(F1,F2))   :- entails(P,F1), entails(P,F2).
entails     (P,  or(F1,F2))   :- entails(P,F1).
entails     (P,  or(F1,F2))   :- entails(P,F2).
not_entails (P1,   b(AG,F))   :- not entails(P2,F), believes(P1,P2,AG).
entails     (P,    b(AG,F))   :- not not_entails(P,b(AG,F)).
```

Entailment

Given a possibility P and a belief formula F.

```
entails      (P,            F)   :- holds(P,F), fluent(F).

entails      (P,       neg(F))   :- not entails(P,F).
entails      (P,  and(F1,F2))    :- entails(P,F1), entails(P,F2).
entails      (P,   or(F1,F2))    :- entails(P,F1).
entails      (P,   or(F1,F2))    :- entails(P,F2).

not_entails  (P1,     b(AG,F))   :- not entails(P2,F), believes(P1,P2,AG).
entails      (P,      b(AG,F))   :- not not_entails(P,b(AG,F)).

not_entails  (P1,    c(AGS,F))   :- not entails(P2,F), reaches(P1,P2,AGS).
entails      (P,     c(AGS,F))   :- not not_entails(P,c(AGS,F)).
```

Ontic actions

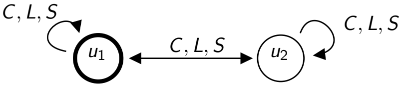
Let *open* be an ontic action such that

- It sets the fluent `opened` to true
- Only ?? and ?? are attentive

Ontic actions

Let *open* be an ontic action such that

- It sets the fluent opened to true
- Only ?? and ?? are attentive



$u_1(\mathcal{F}) = \{head, looking_{Charlie}, looking_{Snoopy}\}$

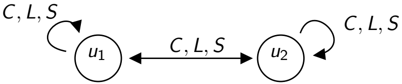
$u_2(\mathcal{F}) = \{looking_{Charlie}, looking_{Snoopy}\}$

Ontic actions

Let *open* be an ontic action such that

- It sets the fluent opened to true
- Only ?? and ?? are attentive

$$\texttt{pw}(\texttt{u}'_{\texttt{i}}) \texttt{ :- pointed}(\texttt{u}_1), \texttt{reaches}(\texttt{u}_1, \texttt{u}_{\texttt{i}}, \texttt{AGS}), \texttt{fully\_obs}(\texttt{AGS}). \quad (i \in \{1,2\})$$



$u_1(\mathcal{F}) = \{head, looking_{Charlie}, looking_{Snoopy}\}$

$u_2(\mathcal{F}) = \{looking_{Charlie}, looking_{Snoopy}\}$

$u'_1(\mathcal{F}) = \{opened, head, looking_{Charlie}, looking_{Snoopy}\}$

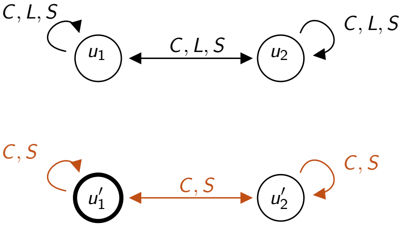
$u'_2(\mathcal{F}) = \{opened, looking_{Charlie}, looking_{Snoopy}\}$

Ontic actions

Let *open* be an ontic action such that

- It sets the fluent opened to true
- Only ?? and ?? are attentive

$$\texttt{believes}(u_i', u_j', AG) \texttt{ :- } \texttt{pw}(u_i'), \texttt{pw}(u_j'), \texttt{fully\_obs}(AG), \texttt{believes}(u_i, u_j, AG), \texttt{pw}(u_i), \texttt{pw}(u_j). \qquad (i, j \in \{1, 2\})$$



$u_1(\mathcal{F}) = \{head, looking_{Charlie}, looking_{Snoopy}\}$

$u_2(\mathcal{F}) = \{looking_{Charlie}, looking_{Snoopy}\}$

$u_1'(\mathcal{F}) = \{opened, head, looking_{Charlie}, looking_{Snoopy}\}$

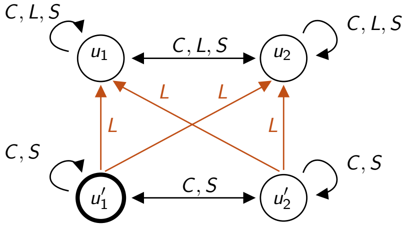
$u_2'(\mathcal{F}) = \{opened, looking_{Charlie}, looking_{Snoopy}\}$

Ontic actions

Let *open* be an ontic action such that

- It sets the fluent opened to true
- Only ?? and ?? are attentive

$$\texttt{believes}(\texttt{u}_\texttt{i}', \texttt{u}_\texttt{j}, \texttt{AG}) \texttt{ :- } \texttt{pw}(\texttt{u}_\texttt{i}'), \texttt{oblivious}(\texttt{AG}), \texttt{believes}(\texttt{u}_\texttt{i}, \texttt{u}_\texttt{j}, \texttt{AG}), \texttt{pw}(\texttt{u}_\texttt{i}), \texttt{pw}(\texttt{u}_\texttt{j}). \qquad (i, j \in \{1, 2\})$$



$u_1(\mathcal{F}) = \{head, looking_{Charlie}, looking_{Snoopy}\}$

$u_2(\mathcal{F}) = \{looking_{Charlie}, looking_{Snoopy}\}$

$u_1'(\mathcal{F}) = \{opened, head, looking_{Charlie}, looking_{Snoopy}\}$

$u_2'(\mathcal{F}) = \{opened, looking_{Charlie}, looking_{Snoopy}\}$

Sensing/Announcement actions

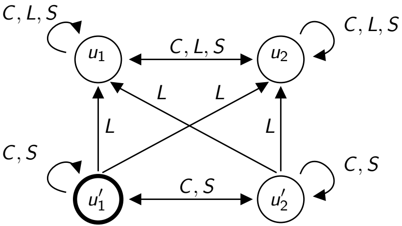
Let *peek* be an ontic action such that

- ?? senses the fluent `heads`
- Only ?? and ?? are attentive

Sensing/Announcement actions

Let *peek* be an ontic action such that

- ?? senses the fluent `heads`
- Only ?? and ?? are attentive



$u_1(\mathcal{F}) = \{head, looking_{Charlie}, looking_{Snoopy}\}$

$u_2(\mathcal{F}) = \{looking_{Charlie}, looking_{Snoopy}\}$

$u'_1(\mathcal{F}) = \{opened, head, looking_{Charlie}, looking_{Snoopy}\}$

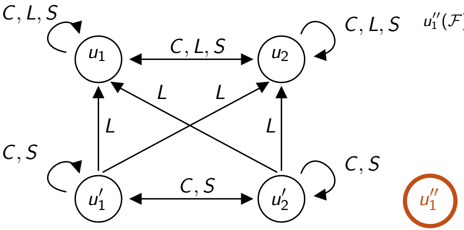
$u'_2(\mathcal{F}) = \{opened, looking_{Charlie}, looking_{Snoopy}\}$

Sensing/Announcement actions

Let *peek* be an ontic action such that

- ?? senses the fluent **heads**
- Only ?? and ?? are attentive

```
pw(u1'') :-  pointed(u1'), reaches(u1', u1', AGS), fully_obs(AGS),
             holds_sensed(u1, peek).
```



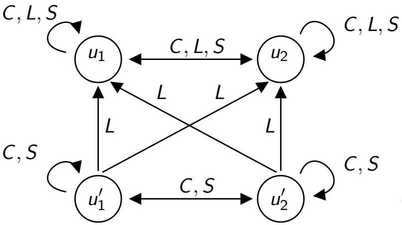
$u_1''(\mathcal{F}) = \{opened, head, looking_{Charlie}, looking_{Snoopy}\}$

Sensing/Announcement actions

Let *peek* be an ontic action such that

- ?? senses the fluent **heads**
- Only ?? and ?? are attentive

pw(u''_2) :- pointed(u'_1), reaches(u'_1, u'_2, AGS), not_oblivious(AGS).



$u''_1(\mathcal{F}) = \{opened, head, looking_{Charlie}, looking_{Snoopy}\}$

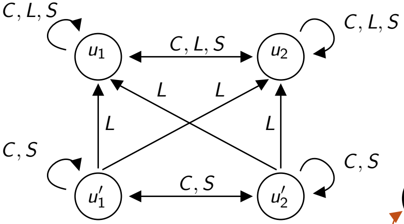
$u''_2(\mathcal{F}) = \{opened, looking_{Charlie}, looking_{Snoopy}\}$

Sensing/Announcement actions

Let *peek* be an ontic action such that

- ?? senses the fluent heads
- Only ?? and ?? are attentive

$$\texttt{believes}(u_i'', u_j'', AG) \texttt{ :- } \texttt{pw}(u_i''), \texttt{pw}(u_j''), \texttt{pw}(u_i'), \texttt{pw}(u_j'), \qquad (i, j \in \{1, 2\})$$
$$\texttt{believes}(u_i', u_j', AG), \texttt{fully\_obs}(AG),$$
$$\texttt{holds\_sensed}(u_i, peek) = \texttt{holds\_sensed}(u_j, peek).$$



$u_1''(\mathcal{F}) = \{opened, head, looking_{Charlie}, looking_{Snoopy}\}$

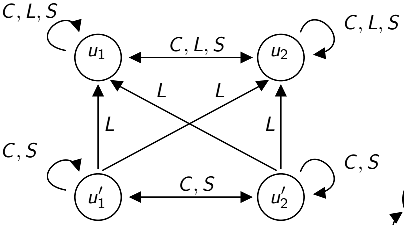
$u_2''(\mathcal{F}) = \{opened, looking_{Charlie}, looking_{Snoopy}\}$

Sensing/Announcement actions

Let *peek* be an ontic action such that

- ?? senses the fluent `heads`
- Only ?? and ?? are attentive

$$\texttt{believes}(\mathtt{u}''_\mathtt{i}, \mathtt{u}''_\mathtt{j}, \mathtt{AG}) \;\texttt{:-}\; \texttt{pw}(\mathtt{u}''_\mathtt{i}), \texttt{pw}(\mathtt{u}''_\mathtt{j}), \texttt{pw}(\mathtt{u}'_\mathtt{i}), \texttt{pw}(\mathtt{u}'_\mathtt{j}), \texttt{believes}(\mathtt{u}'_\mathtt{i}, \mathtt{u}'_\mathtt{j}, \mathtt{AG}), \texttt{partially\_obs}(\mathtt{AG}). \qquad (i, j \in \{1, 2\})$$



$u''_1(\mathcal{F}) = \{opened, head, looking_{Charlie}, looking_{Snoopy}\}$

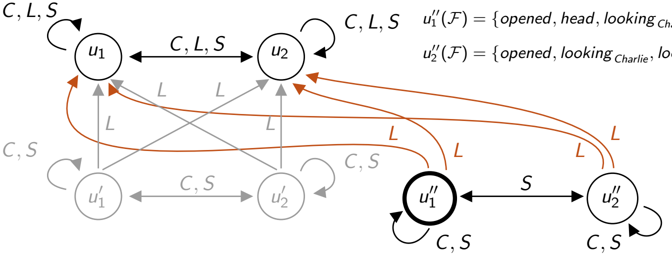
$u''_2(\mathcal{F}) = \{opened, looking_{Charlie}, looking_{Snoopy}\}$

Sensing/Announcement actions

Let *peek* be an ontic action such that

- ?? senses the fluent **heads**
- Only ?? and ?? are attentive

$$\texttt{believes}(\texttt{u}''_\texttt{i}, \texttt{u}_\texttt{j}, \texttt{AG}) \texttt{ :- } \texttt{pw}(\texttt{u}''_\texttt{i}), \texttt{pw}(\texttt{u}'_\texttt{i}), \texttt{pw}(\texttt{u}_\texttt{j}), \texttt{believes}(\texttt{u}'_\texttt{i}, \texttt{u}_\texttt{j}, \texttt{AG}), \texttt{oblivious}(\texttt{AG}). \quad (i, j \in \{1, 2\})$$



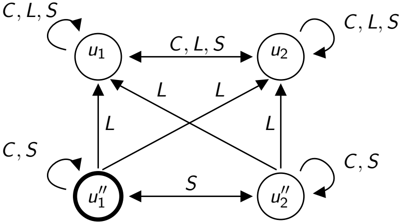
$$u''_1(\mathcal{F}) = \{opened, head, looking_{Charlie}, looking_{Snoopy}\}$$

$$u''_2(\mathcal{F}) = \{opened, looking_{Charlie}, looking_{Snoopy}\}$$

Sensing/Announcement actions

Let *peek* be an ontic action such that

- ?? senses the fluent `heads`
- Only ?? and ?? are attentive



$u_1(\mathcal{F}) = \{head, looking_{Charlie}, looking_{Snoopy}\}$

$u_2(\mathcal{F}) = \{looking_{Charlie}, looking_{Snoopy}\}$

$u''_1(\mathcal{F}) = \{opened, head, looking_{Charlie}, looking_{Snoopy}\}$

$u''_2(\mathcal{F}) = \{opened, looking_{Charlie}, looking_{Snoopy}\}$

Correctness w.r.t. $m\mathcal{A}^\rho$

Let u, v be two possibilities and ψ be a belief formula.

Entailment correctness

For each u, we have that $\forall\ \psi \quad u \models_\Phi \psi \;$ iff $\; u \models_\Gamma \psi$.

Initial state generation correctness

For each u, v such that u is the initial state in $m\mathcal{A}^\rho$ and v is the initial state in PLATO then $\forall\ \psi \quad u \models_\Phi \psi \;$ iff $\; v \models_\Gamma \psi$.

Transition function correctness

Let a be an action instance. For each u, v such that $\forall\ \psi \quad u \models_\Phi \psi$ iff $v \models_\Gamma \psi$, then $\forall\ \psi \quad \Phi(a, u) \models_\Phi \psi \;$ iff $\; \Gamma(a, v) \models_\Gamma \psi$.

Experimental evaluation

SC: $\vert\mathcal{AG}\vert = 9$, $\vert\mathcal{F}\vert = 12$, $\vert\mathcal{A}\vert = 14$				
L	`many`	`frumpy`	K-BIS	P-MAR
4	.24	.24	.03	**.007**
6	2.56	2.49	.16	**.04**
8	36.79	38.34	4.23	**.30**
9	204.52	146.343	5.79	**.83**
10	`TO`	839.27	7.36	**1.78**

Gr: $\vert\mathcal{AG}\vert = 3$, $\vert\mathcal{F}\vert = 9$, $\vert\mathcal{A}\vert = 24$				
L	Total	Ground	Solve	Atoms
3	.97	.60	.36	28’615
4	4.25	2.24	2.01	42’022
5	32.83	2.52	30.31	71’482
6	211.69	5.27	206.41	140’305
7	1066.80	16.94	1049.86	302’623

Experimental evaluation

SC: $\vert\mathcal{AG}\vert = 9$, $\vert\mathcal{F}\vert = 12$, $\vert\mathcal{A}\vert = 14$				
L	`many`	`frumpy`	K-BIS	P-MAR
4	.24	.24	.03	**.007**
6	2.56	2.49	.16	**.04**
8	36.79	38.34	4.23	**.30**
9	204.52	146.343	5.79	**.83**
10	`TO`	839.27	7.36	**1.78**

Gr: $\vert\mathcal{AG}\vert = 3$, $\vert\mathcal{F}\vert = 9$, $\vert\mathcal{A}\vert = 24$				
L	Total	Ground	Solve	Atoms
3	.97	.60	.36	28'615
4	4.25	2.24	2.01	42'022
5	32.83	2.52	30.31	71'482
6	211.69	5.27	206.41	140'305
7	1066.80	16.94	1049.86	302'623

	CC_1: $\vert\mathcal{AG}\vert = 2$, $\vert\mathcal{F}\vert = 10$, $\vert\mathcal{A}\vert = 16$				**CC_2**: $\vert\mathcal{AG}\vert = 3$, $\vert\mathcal{F}\vert = 13$, $\vert\mathcal{A}\vert = 24$			
L	`single`	`multi`	K-BIS	P-MAR	`single`	`multi`	K-BIS	P-MAR
3	48.74	6.52	.08	**.02**	153.76	14.07	.13	**.03**
4	188.32	8.74	.16	**.03**	`TO`	28.02	.54	**.10**
5	`TO`	7.68	1.14	**.16**	`TO`	16.13	4.89	**.60**
6	1222.67	10.83	4.42	**0.64**	`TO`	14.84	12.66	**1.71**
7	`TO`	30.08	16.06	**2.61**	`TO`	56.48	142.06	**12.37**

Chapter 5

Conclusions

Conclusions

- Exploited a declarative approach to implement Multi-Agent Epistemic Planning
- Improved readability and code maintenance
- Straightforward semantical adaptations
- Results comparable to the imperative approach
- Formal proof of correctness

Future works

- Enhancement of the entailment rules
- Implementation of heuristics
- Formal proof of equivalence between $m\mathcal{A}^*$ and $m\mathcal{A}^\rho$
- We are using PLATO to implement novel concepts in MEP, such as trust, lies and misconceptions

The end

Thank You
for the attention

References I

Ohter Planners

To the best of our knowledge ?? is the only *comprehensive* epistemic multi-agent planner.

Other planners with the best results in the literature are:

- ?? **muise2015planning**: translates into classical planning. Only deals with a finite level of nested beliefs.
- ?? **huang2017general**: does not support dynamic common knowledge.

Domains I

- *Assembly Line* (**AL**): two agents are responsible for processing a different part of a product. They can fail in processing their part and inform the other of the status of her task. The agents decide to *assemble* the product or *restart*. Goal: the agents must assemble the product. We change the *depth* of the belief formulae.
- *Coin in the Box* (**CB**). $n \geq 3$ agents are in a room. There is a closed box containing a coin. None of the agents know the coin position. One agent has the key. An agent may look inside the box to sense the state of the coin and also share the result.

Domains II

- *Collaboration and Communication* (**CC**). $n \geq 2$ agents move along a corridor with $k \geq 2$ rooms in which $m \geq 1$ boxes can be located. Agents can determine if a certain box is in the room they are in. They can communicate information about the boxes' position. Agents may move only to adjacent rooms.
- *Grapevine*. $n \geq 2$ agents are located in $k \geq 2$ rooms. Each agent ag knows a "secret" (`s_ag`). Agents can move to an adjacent room and share their secret within the same room.
- *Selective Communication* (**SC**). $n \geq 2$ agents within one of the $k \geq 2$ rooms in a corridor. Agents can move to an adjacent room. In only one of the rooms, agents may acquire some information `q` and may communicate it to others.

Finitary S5 Theories

Finitary **S5**-theory **son2014finitary**

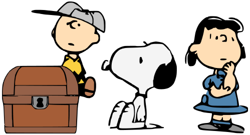
Let ϕ be a fluent formula and let $\mathsf{i} \in \mathcal{AG}$ be an agent. A *finitary S5-theory* is a collection of formulae of the form:

$$(i)\ \phi \quad (ii)\ C\ \phi \quad (iii)\ C\ (B_{\mathsf{i}}\phi \vee B_{\mathsf{i}}\neg\phi) \quad (iv)\ C\ (\neg B_{\mathsf{i}}\phi \wedge \neg B_{\mathsf{i}}\neg\phi)$$

Each fluent $\mathsf{f} \in \mathcal{F}$ must appear in at least one of the formulae (*ii*)–(*iv*) (for at least one agent $\mathsf{i} \in \mathcal{AG}$).

A finitary **S5**-theory has *finitely many* S5-models up to equivalence.

Initial state generation



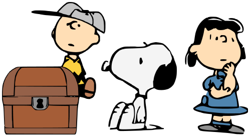
Given

- $\mathcal{AG} = \{??, ??, ??\}$
- $\mathcal{F} \quad = \{\texttt{opened}, \texttt{head}, \texttt{looking}_{\text{ag}}\}$ ag $\in \mathcal{AG}$

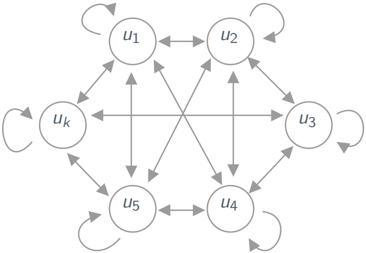
Initial state generation

Given

- $\mathcal{AG} = \{??, ??, ??\}$
- $\mathcal{F} \quad = \{\texttt{opened}, \texttt{head}, \texttt{looking}_{\text{ag}}\}$ ag $\in \mathcal{AG}$

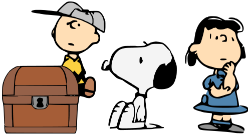


Consider a formula of a finitary **S5** theory.



Formula type:

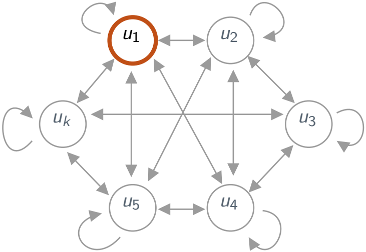
Initial state generation



Given

- $\mathcal{AG} = \{??, ??, ??\}$
- $\mathcal{F} \quad = \{\texttt{opened}, \texttt{head}, \texttt{looking}_{\text{ag}}\}$ ag $\in \mathcal{AG}$

Consider a formula of a finitary **S5** theory.



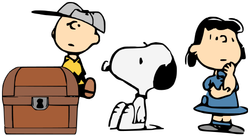
Formula type:

(i) ϕ

Initial state generation

Given

- $\mathcal{AG} = \{??, ??, ??\}$
- $\mathcal{F} \quad = \{\texttt{opened}, \texttt{head}, \texttt{looking}_{\texttt{ag}}\}$ ag $\in \mathcal{AG}$



Consider a formula of a finitary **S5** theory.

$u_1(\mathcal{F}) = \{head, looking_{Charlie}, looking_{Snoopy}\}$

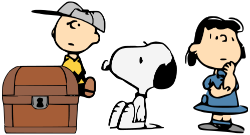
$u_2(\mathcal{F}) = \{looking_{Charlie}, looking_{Snoopy}\}$

Formula type:

(i) ϕ

(ii) $C\ \phi$

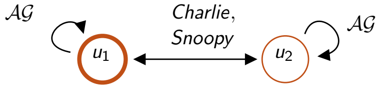
Initial state generation



Given

- $\mathcal{AG} = \{??, ??, ??\}$
- $\mathcal{F} \quad = \{\texttt{opened}, \texttt{head}, \texttt{looking}_{\text{ag}}\}$ ag $\in \mathcal{AG}$

Consider a formula of a finitary **S5** theory.



$u_1(\mathcal{F}) = \{head, looking_{Charlie}, looking_{Snoopy}\}$

$u_2(\mathcal{F}) = \{looking_{Charlie}, looking_{Snoopy}\}$

Formula:

$C\ (B_{\text{Lucy}} head \vee B_{\text{Lucy}} \neg head)$

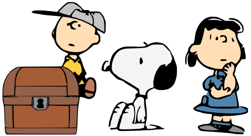
Formula type:

(i) ϕ

(ii) $C\ \phi$

(iii) $C\ (B_{\text{i}}\phi \vee B_{\text{i}}\neg\phi)$

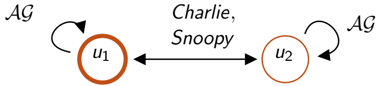
Initial state generation



Given

- $\mathcal{AG} = \{??, ??, ??\}$
- $\mathcal{F} \quad = \{\texttt{opened}, \texttt{head}, \texttt{looking}_{\text{ag}}\}$ ag $\in \mathcal{AG}$

Consider a formula of a finitary **S5** theory.



$u_1(\mathcal{F}) = \{head, looking_{Charlie}, looking_{Snoopy}\}$

$u_2(\mathcal{F}) = \{looking_{Charlie}, looking_{Snoopy}\}$

Formula type:

(i) ϕ

(ii) $C\ \phi$

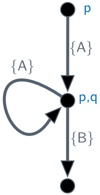
(iii) $C\ (B_i\phi \vee B_i\neg\phi)$

(iv) $C\ (\neg B_i\phi \wedge \neg B_i\neg\phi)$

From Possibilities to Kripke Structures

Considering a possibility

A possibility

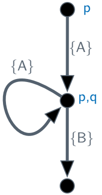


From Possibilities to Kripke Structures

Considering a possibility

Can be expressed as a *system of equations*

A possibility



Its system of equation

$$\begin{cases} w(p) = 1 & w(q) = 0 \\ v(p) = 1 & v(q) = 1 \\ u(p) = 0 & u(q) = 0 \\ w(A) = \{v\} & w(B) = \{\emptyset\} \\ v(A) = \{v\} & v(B) = \{u\} \\ u(A) = \{\emptyset\} & u(B) = \{\emptyset\} \end{cases}$$

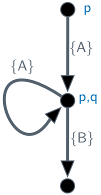
From Possibilities to Kripke Structures

Considering a possibility

Can be expressed as a *system of equations*

Systems of equations have unique solutions

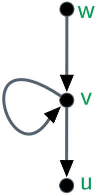
A possibility



Its system of equation

$$\begin{cases} w(p) = 1 & w(q) = 0 \\ v(p) = 1 & v(q) = 1 \\ u(p) = 0 & u(q) = 0 \\ w(A) = \{v\} & w(B) = \{\emptyset\} \\ v(A) = \{v\} & v(B) = \{u\} \\ u(A) = \{\emptyset\} & u(B) = \{\emptyset\} \end{cases}$$

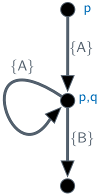
The solution



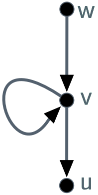
From Possibilities to Kripke Structures

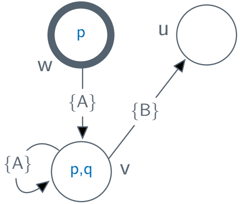
Considering a possibility
- Can be expressed as a *system of equations*
  - Systems of equations have unique solutions
    - The solution decorates a Kripke structure

A possibility	Its system of equation	The solution	Relative Kripke Structure



$$\begin{cases} w(p) = 1 & w(q) = 0 \\ v(p) = 1 & v(q) = 1 \\ u(p) = 0 & u(q) = 0 \\ w(A) = \{v\} & w(B) = \{\emptyset\} \\ v(A) = \{v\} & v(B) = \{u\} \\ u(A) = \{\emptyset\} & u(B) = \{\emptyset\} \end{cases}$$





From Possibilities to Kripke Structures

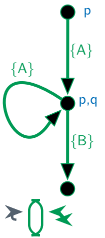
Considering a possibility

Can be expressed as a *system of equations*

Systems of equations have unique solutions

The solution decorates a Kripke structure

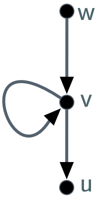
A possibility



Its system of equation

$$\begin{cases} w(p) = 1 & w(q) = 0 \\ v(p) = 1 & v(q) = 1 \\ u(p) = 0 & u(q) = 0 \\ w(A) = \{v\} & w(B) = \{\emptyset\} \\ v(A) = \{v\} & v(B) = \{u\} \\ u(A) = \{\emptyset\} & u(B) = \{\emptyset\} \end{cases}$$

The solution



Relative Kripke Structure