DELPHIC: PRACTICAL DEL PLANNING VIA POSSIBILITIES

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- Agents might do something depending on what they know
- Cooperative setting: agents want to reach a common goal
- Centralized setting: a single omniscient entity (the planner) is responsible for finding a solution

Example (Coin in the Box)

Initial situation. Anne, Bob and Carl are in the same room. A coin placed inside a closed box. Everybody knows that the box is closed (c), but no one knows the position of the coin.

There are two possible situations:

- The coin lies heads up (h), and
- The coin lies tails up $(\neg h)$.

Goals can include epistemic conditions:

- Anne knows/believes that *h*,
- Bob knows/believes that Anne knows/believes whether h or not,
- Carl knows/believes that Anne does not know/believe whether h,
- Both Bob and Carl do not know/believe whether h.

DYNAMIC EPISTEMIC LOGIC

The Language

Let \mathcal{P} be a finite set of propositional atoms and $\mathcal{AG} = \{1, \ldots, n\}$ a finite set of agents.

Definition (Language $\mathcal{L}_{\mathcal{P},\mathcal{A}\mathcal{G}}$)

$$\varphi ::= p \mid \neg \varphi \mid \varphi \land \varphi \mid \Box_i \varphi,$$

Example (Coin in the Box)

Let $\mathcal{P} = \{c, h\}$ and $\mathcal{AG} = \{Anne, Bob, Carl\}$. We can state the conditions of our example as follows:

Initial conditions:

• $\bigwedge_{i \in AG} \Box_i c$

Goal conditions:

■ □_{Anne}h

- $\blacksquare \square_{Bob}(\square_{Anne} h \lor \square_{Anne} \neg h)$
- $\square \square_{Carl} (\neg \square_{Anne} h \land \neg \square_{Anne} \neg h)$
- $\bigwedge_{i \in \{\text{Bob}, Carl\}} (\neg \Box_i h \land \neg \Box_i \neg h)$



Figure: Initial state.

Epistemic states (pointed Kripke models):

- Worlds: possible situations
- Relations: what agents consider to be possible
- Valuation: what is considered to be true in each world
- Designated worlds: actual situations



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Definition (Truth)

$$\begin{array}{ll} (M,w) \models p & \text{iff} & w \in V(p) \\ (M,w) \models \neg \phi & \text{iff} & (M,w) \not\models \phi \\ (M,w) \models \phi \land \psi & \text{iff} & (M,w) \models \phi \text{ and } (M,w) \models \psi \\ (M,w) \models \Box_i \phi & \text{iff} & \forall v \text{ if } wR_i v \text{ then } (M,v) \models \phi \end{array}$$





Figure: Anne opens the box while only Bob is looking (Carl is distracted).

Actions (pointed event models):

- Events: what might happen relatively to some agents' perspective
- Relations: akin to those of epistemic models
- Preconditions: what is needed for an event to occur
- Postconditions: how an event changes a world
- Designated events: what actually happens







Product update:



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Product update:

$$(w_1, e_2): h, c$$
 $(w_2, e_2): c$







Product update:





Notice that w_1 (resp., w_2) and (w_1, e_2) (resp., (w_2, e_2)) encode the same information, but they are distinct objects!

DELPHIC

DEL-planning with a Possibility-based Homogeneous Information Characterisation:

- Epistemic models represented by possibilities
- Event models represented by eventualities
- New semantics for actions: union update

A **possibility** u is a function that assigns to each atom $p \in \mathcal{P}$ a truth value $u(p) \in \{0, 1\}$ and to each agent $i \in A\mathcal{G}$ a set of possibilities u(i).

Definition (Possibility spectrum)

A possibility spectrum is a non-empty set $U = \{u_1, \ldots, u_k\}$ of designated possibilities.

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Intuitively:

- u(p) specifies the truth value of the atom p (plays the role of the valuation function)
- u(i) is the set of all the worlds that agent i considers possible in u (plays the role of the accessibility relations)
- A possibility spectrum plays the role of the designated worlds

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$$U = \{w_1\}, \text{ where:}$$

• $w_1(h) = w_1(c) = 1$
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Example



 $U = \{w_1\}, \text{ where:}$ $w_1(h) = w_1(c) = 1$ $w_1(A) = w_1(B) = w_1(C) = \{w_1, w_2\}$ $w_2(h) = 0 \text{ and } w_2(c) = 1$ $w_2(A) = w_2(B) = w_2(C) = \{w_1, w_2\}$

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$$\begin{array}{ll} \mathsf{u} \models \rho & \text{iff} & \mathsf{u}(p) = 1 \\ \mathsf{u} \models \neg \phi & \text{iff} & \mathsf{u} \not\models \phi \\ \mathsf{u} \models \phi \land \psi & \text{iff} & \mathsf{u} \models \phi \text{ and } \mathsf{u} \models \psi \\ \mathsf{u} \models \Box_i \phi & \text{iff} & \forall \mathsf{v} \text{ if } \mathsf{v} \in \mathsf{u}(i) \text{ then } \mathsf{v} \models \phi \end{array}$$

Finally, $U \models \varphi$ iff $v \models \varphi$, for all $v \in U$.

Let $pre \notin \mathcal{P}$ be a fresh atom and let $\mathcal{P}' = \mathcal{P} \cup \{pre\}$.

Definition (Eventuality)

An eventuality e is a function that assigns to each atom $p' \in \mathcal{P}'$ a formula $e(p') \in \mathcal{L}_{\mathcal{P},\mathcal{AG}}$ and to each agent $i \in \mathcal{AG}$ a set of eventualities e(i).

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- e(*pre*) specifies precondition of e
- e(p) specifies postcondition of p in e
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$$E = \{e_1\}, \text{ where:}$$

$$e_1(pre) = c, e_1(h) = h \text{ and } e_1(c) = \bot$$

$$e_1(A) = e_1(B) = \{e_1\} \text{ and } e_1(C) = \{e_2\}$$

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, $e_1(h) = h$ and $e_1(c) = \bot$

•
$$e_1(A) = e_1(B) = \{e_1\} \text{ and } e_1(C) = \{e_2\}$$

•
$$e_2(pre) = \top$$
, $e_2(h) = h$ and $e_2(c) = c$

$$e_2(A) = e_2(B) = e_2(C) = \{e_2\}$$
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An eventuality e is **applicable** in a possibility u iff $u \models e(pre)$.

Definition (Union Update)

The **union update** of a possibility u with an applicable eventuality e is the possibility $u' = u \otimes e$, such that:

- $\mathbf{u'}(p) = 1$ iff $\mathbf{u} \models \mathbf{e}(p)$
- $u'(i) = \{v \otimes f \mid v \in u(i), f \in e(i) \text{ and } v \models f(pre)\}$

The **union update** of a possibility spectrum U with an eventuality spectrum E is the possibility spectrum $U \boxtimes E = \{u \boxtimes e \mid u \in U, e \in E \text{ and } u \models e(pre)\}.$

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$$\mathsf{U} \boxtimes \mathsf{E} = \{\mathsf{w}_1 \boxtimes \mathsf{e}_1\} = \{\mathsf{w}_1^1\}, \text{ where:}$$



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$$\begin{split} & \boxtimes \mathsf{E} = \{\mathsf{w}_1 \boxtimes \mathsf{e}_1\} = \{\mathsf{w}_1^1\}, \text{ where:} \\ & = \mathsf{w}_1^1(c) = 0 \text{ and } \mathsf{w}_1^1(h) = 1 \\ & = \mathsf{w}_1^1(A) = \mathsf{w}_1^1(B) = \{\mathsf{w}_1 \boxtimes \mathsf{e}_1, \mathsf{w}_2 \boxtimes \mathsf{e}_1\} \text{ and } \mathsf{w}_1^1(C) = \{\mathsf{w}_1 \boxtimes \mathsf{e}_2, \mathsf{w}_2 \boxtimes \mathsf{e}_2\} \end{split}$$

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$$= w_1^2 = w_1 \text{ and } w_2^2 = w_2 \text{ (we can reuse old information!)}$$

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DISCUSSION

DELPHIC overcomes some shortcomings of DEL:

- Does not reuse old information (as shown before)
- Blind cross-product: may result into unreachable information
 - \rightarrow World (w_2 , e_2) is redundant: it is not reachable from a designated world



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Example

$$A, B \qquad A, B \qquad A = 0 \qquad$$

Let $U = \{w_1\}$ and $E = \{e_1, e_2\}$, where: • $w_1(p)=1$ and $w_1(A)=w_1(B)=\{w_1, w_2\}$ • $w_2(p)=0$ and $w_2(A)=w_2(B)=\{w_1, w_2\}$

e₁(pre)= p, e₁(p)=p and e₁(A)=e₁(B)={e₁}
 e₂(pre)=¬p, e₂(p)=p and e₂(A)=e₂(B)={e₂}

DELPHIC overcomes some shortcomings of DEL:

- Does not reuse old information (as shown before)
- Blind cross-product: may result into unreachable information
 - \rightarrow World (*w*₂, *e*₂) is redundant: it is not reachable from a designated world

Example

In DELPHIC every possibility is reachable: $U \otimes E = \{w_1 \otimes e_1\} = \{w_1\}$.

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 - $\rightarrow\,$ Possibilities allow for a more compact representation

We can exploit this property in implementations of tools:

- Possibilities have already been proved to provide more efficient implementations
- Epistemic planner EFP 2.0 [Fab+20]: relies on a framework called *m*A* [Bar+15], which is a fragment of DEL

EXPERIMENTAL EVALUATION

- We implemented DELPHIC and the traditional Kripke-based DEL semantics.
- We used the well-known declarative language **ASP** (*Answer Set Programming*). → Fair and transparent comparison.
- We compared the two ASP models both in terms of space and time.
 - $\rightarrow\,$ We used benchmarks found in the literature.

You can find our implementation here: github.com/a-burigana/delphic_asp.



DELPHIC vs. Kripke



w14 (w10, sig) looking a, looking b, looking c, opened, tail

DELPHIC vs. Kripke



CONCLUSIONS

- We introduced DELPHIC, an alternative semantics for Dynamic Epistemic Logic.
- The DELPHIC framework is **equivalent** to the Kripke-based one.
- We empirically showed that DELPHIC outperforms the traditional Kripke-based semantics both in space and time.

Future works:

■ Implement DELPHIC in more competitive solvers (*e.g.*, *EFP* [Fab+20]).

THANK YOU Questions?