A SEMANTIC APPROACH TO DECIDABILITY IN EPISTEMIC PLANNING

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DYNAMIC EPISTEMIC LOGIC

Epistemic planning is an enrichment of automated planning where the concept of **knowledge/belief** is taken into account.

Example (Coin in the Box)

Initial situation. Anne, Bob and Carl are in a room. A coin placed inside a closed box. Everybody knows that the box is closed (c), but no one knows the position of the coin.

There are two possible situations:

- The coin lies heads up (h), and
- The coin lies tails up ($\neg h$).

Goals can include **epistemic conditions**:

- Anne knows/believes that h,
- Bob knows/believes that Anne knows/believes whether h or not,
- Carl knows/believes that Anne does not know/believe whether h,
- Both Bob and Carl do not know/believe whether h.

Dynamic Epistemic Logic

Let $\mathcal{P}$ be a finite set of propositional atoms and $\mathcal{AG} = \{1, \dots, n\}$ a finite set of agents.

Definition (Language $\mathcal{L}^{C}_{\mathcal{P},\mathcal{AG}}$)

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \Box_i \varphi \mid C_G \varphi$$

Example (Coin in the Box)

Let $\mathcal{P} = \{c, h\}$ and $\mathcal{AG} = \{Anne, Bob, Carl\}$. We can state the conditions of our example as follows:

Initial conditions:

- $\bigwedge_{i \in \mathcal{AG}} (\neg\Box_i h \wedge \neg\Box_i \neg h)$
- $C_{\mathcal{AG}}\, c$

Goal conditions:

- $\Box_{Anne} h$
- $\Box_{Bob}(\Box_{Anne} h \vee \Box_{Anne} \neg h)$
- $\Box_{Carl}(\neg\Box_{Anne} h \wedge \neg\Box_{Anne} \neg h)$
- $\bigwedge_{i \in \{Bob, Carl\}} (\neg\Box_i h \wedge \neg\Box_i \neg h)$

A Very Expressive Semantics

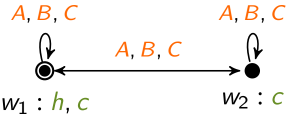

Figure: Initial state.

Epistemic states (pointed Kripke models):

- Uncertainty
- Higher order knowledge/belief
- Nondeterminism

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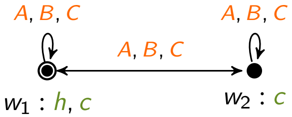

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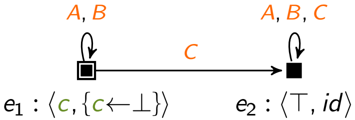

Figure: Anne opens the box while only Bob is looking (Carl is distracted).

Epistemic states (pointed Kripke models):

- Uncertainty
- Higher order knowledge/belief
- Nondeterminism

Actions (pointed event models):

- Epistemic and ontic change
- Partial observability
- Nondeterminism

The Price of Expressiveness

Notoriously, the **(epistemic) plan existence problem** in the logic **$S5_n$** is **undecidable**.

→ Reduction to halting problem of Turing machines [BA11] and 2-counter machines [AB13].

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Many existing approaches to decidability rely on **syntactical restrictions** (modal depth):

d_{pre}	d_{post}	Plan existence problem
0	-	**PSPACE-complete** [CMS16]
1	-	**Unknown** [CMS16]
2	-	**Undecidable** [CMS16]
0	0	**Decidable** [YWL13; AMP14]
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Let's try a different approach!

THE SEMANTIC APPROACH

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We switch our attention to the **logic** of the plan existence problem.

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Let's push the envelope!

The Knowledge Commutativity Axiom

Agents have too much reasoning power: they can **reason unboundedly** about each other's knowledge.

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Knowledge Commutativity

$$\mathbf{C} \quad \Box_i \Box_j \varphi \to \Box_j \Box_i \varphi$$

We call $\mathbf{C\text{-}S5}_n$ the logic $S5_n$ augmented with axiom **C**.

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- Principle of *commutativity* in the knowledge that agents have about the knowledge of others.
- Reasonable assumption in several *cooperative multi-agent planning tasks* [journals/csur/Torreno2017] where agents are able to communicate or monitor each other.

Common Knowledge in C-S5_n

C-S5_n admits a finitary non-fixpoint characterization of common knowledge:

Theorem

Let $G = \{i_1, \ldots, i_m\}$, with $G \subseteq \mathcal{AG}$ and $m \geqslant 2$. In C-S5_n, for any φ, the formula

$$\Box_{i_1} \ldots \Box_{i_m} \varphi \leftrightarrow C_G \varphi$$

is a theorem.

Often, common knowledge is regarded as "too strong". Instead, in C-S5_n the power of common knowledge is more controlled.

- Application to the **Coordinated Attack Problem**: the two generals realize that they can not achieve common knowledge about the plan for the attack.

Size of Epistemic States and Decidability in C-S5$_n$

A very helpful property of C-S5$_n$-states:

Lemma

*Let (M, W_d) be a bisimulation-contracted C-S5$_n$-state, with $M = (W, R, V)$. Then, $|W|$ **is bounded in** n **and** $|\mathcal{P}|$.*

This entails that there exist **finitely many C-S5$_n$-states** (modulo bisimulation-contraction). We can perform a BFS visit.

Theorem

*The plan existence problem in **C-S5$_n$** is **decidable**.*

Epistemic Planning Systems

Two well-known systems fall under the logic C-S5_n: the **S5_n-fragment of $m\mathcal{A}^*$** and the one from Kominis and Geffner (KG).

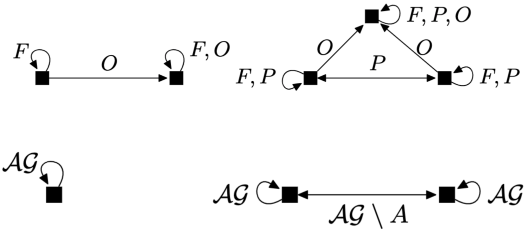

Figure: The systems $m\mathcal{A}^*$ (top) and **KG** (bottom).

Corollary

*The plan existence problem in **S5_n**-$m\mathcal{A}^*$ and KG is **decidable**.*

GENERALIZED KNOWLEDGE COMMUTATIVITY

b-Commutativity

Let $b > 1$ be a fixed integer constant:

b-Commutativity

$$\mathbf{C}^b \quad (\Box_i \Box_j)^b \varphi \to (\Box_j \Box_i)^b \varphi$$

We call **$\mathbf{C}^b$-S5$_n$** the logic S5$_n$ augmented with axiom $\mathbf{C}^b$.

Theorem

For any $b > 1$, the plan existence problem in ***C^b-S5$_2$*** *is* ***decidable****.*

Theorem

For any $n > 2$ and $b > 1$, the plan existence problem in ***C^b-S5$_n$*** *is* ***undecidable****.*

Weak Commutativity

Let $1 < \ell \leqslant n$ be a fixed integer constant, let $\langle i_1, \ldots, i_\ell \rangle$ be a repetition-free sequence of agents and let π be any of its permutations:

Weak Commutativity

$$\mathbf{wC}_\ell \quad \Box_{i_1} \ldots \Box_{i_\ell} \varphi \to \Box_{\pi_{i_1}} \ldots \Box_{\pi_{i_\ell}} \varphi$$

We call **$\mathbf{wC}_\ell$-$\mathbf{S5}_n$** the logic $\mathrm{S5}_n$ augmented with axiom $\mathbf{wC}_\ell$ (for all π).

Theorem

*For any $n > 1$ and $1 < \ell \leqslant n$, the plan existence problem in **$\mathbf{wC}_\ell$-$\mathbf{S5}_n$** is **decidable**.*

Quick Recap

Logic	Plan existence problem
K_n, KT_n, $K4_n$, $K45_n$, $S4_n$, $S5_n$	**Undecidable** [AB13]
C^b-$S5_n$ $(n > 2)$	**Undecidable**
C^b-$S5_2$	**Decidable**
wC_ℓ-$S5_n$	**Decidable**
C-$S5_n$	**Decidable**

CONCLUSIONS

Conclusions and Future Works

To summarize:

- We proposed a novel **semantic approach** to decidability in DEL-planning.
- We showed how one can effectively obtain **decidable fragments** by augmenting the logic $S5_n$ with interaction axioms.
- We showed that two well-known epistemic planning systems fall within our logic, hence proving their decidability.

Future works:

- Analyze complexity of DEL-planning under commutativity.
- Explore more axioms, both on top of $S5_n$ and $KD45_n$.

THANK YOU

Questions?